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Essays in industrial organization

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ESSAYS IN INDUSTRIAL ORGANIZATION

by

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requirements for the Doctor of Philosophy
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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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ABSTRACT

This thesis is composed by three different studies on oligopolies. The first chapter is on oligopolies with perfect complements; the second chapter studies oligopolies with positive network effects and incompatible networks, and the last one deals with a polluting duopoly subject to environmental regulation.

Specifically, the first chapter provides a thorough characterization of the properties of Cournot's complementary monopoly model (or oligopoly with perfect complements) in a general setting, including existence, uniqueness and the comparative statics effects of entry. As such, this serves to unify various results from the extant literature that have typically been derived with limited generality.

Several studies have suggested that Cournot's complementary monopoly model is the dual problem to the standard Cournot oligopoly model. This result crucially relies on the assumption that the firms have no production costs. The first chapter shows that if the production costs of the firms are different from zero, the nice duality between these two oligopoly settings breaks down. One implication of this breakdown is that, in contrast to the Cournot model, oligopoly with perfect complements can be a game of strategic complements in a global sense even in the presence of production costs.

The second chapter models symmetric oligopolies with positive network effects where each firm has its own proprietary network. That is, each firm's network is incompatible with that of its rivals. This chapter provides minimal conditions for

the existence of (non-trivial) equilibrium in a general setting; in this model, the equilibria may be either symmetric or asymmetric. For the symmetric equilibria, this chapter analyzes the comparative statics effects of entry. In addition, it compares the equilibrium outcomes of oligopoly markets with compatible and incompatible networks. It shows that firms with compatible networks produce higher quantities than firms with incompatible networks. However, the relationship between prices, profits and consumer surplus is ambiguous, but social welfare is always higher in markets with completely compatible networks.

Finally, the third chapter analyzes the incentives to invest in R&D under two environmental policy instruments: the emission and performance standards, in a Cournot model of competition between two symmetric firms. These firms are subject to environmental regulations as their production of a homogeneous good entails pollution. Unlike a few models of output market available in the literature, this approach does not measure the environmental incentives using firms' aggregate cost savings. Instead, it compares the levels of social welfare obtained under both policy instruments. From the derived subgame perfect equilibria of the two games, each game associated with a different instrument, this chapter shows that social welfare under performance standard dominates that under emission standard. It also finds that further comparisons, in particular, the comparison of the investment in R&D, are ambiguous and not aligned with the welfare comparison.

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CHAPTER 1
ON COURNOT'S THEORY OF OLIGOPOLY WITH PERFECT COMPLEMENTS

1.1 Preliminaries

1.1.1 Introduction

It is well known that Cournot's (1838) pioneering book set the stage for a major paradigm shift in economic theory. In a more direct manner, it initiated the formal study of imperfectly competitive markets and provided an overly precocious foretaste of game theory. While his basic model of quantity competition amongst few firms became a workhorse of applied microeconomics and remains one of the dominant models of partial equilibrium analysis, his other oligopoly model has remained only modestly known even to this day. Cournot's complementary monopoly model refers to a market with the following features. Consumers have a downward-sloping demand for a final product or a system that can only be put together after the purchase of n different components, each of which is sold exclusively by a monopoly supplier. Any subset of components other than the full set has no value in itself for any consumer. The n components constitute thus perfect complements, and none of them possesses any substitutes. The only meaningful demand is thus for the overall system or final product, and the relevant price for consumers is the sum of all the prices paid for all the n components.

The presence of a group of monopolists selling goods that are perfect comple-

BASED ON JOINT WORK WITH RABAH AMIR.

ments probably explains Cournot's original name for the model. Nonetheless, along the lines of modern game theory, this setting is more aptly referred to as oligopoly with perfect complements, since this explicitly recognizes the strategic interdependence between the "monopolists". (In this paper, we shall use either terminology to refer to this model.)

Two historical settings have motivated the conception of this model. The first, put forth by Cournot (1838) himself is the production of brass, the production of which requires two inputs, copper and zinc at the same time, each supplied by a different monopolist. The demand function here stands for the input demand by the producer for the two inputs (ordered in a one-to-one ratio). Another seminal work in oligopoly with perfect complements was developed independently by Ellet in 1839 (Ellet, 1966). The setting that inspired him dealt with how two different individuals who own consecutive segments of a canal decide their tolls to shippers. We shall discuss a number of different applications of this model throughout the paper.

In an early study, Sonnenschein (1968) showed that Cournot's two theories, the standard Cournot oligopoly and oligopoly with perfect complements, are formally equivalent theories when the costs of production of the firms are zero. Indeed, there is a perfect duality between the revenue functions of the two models, with the decision variables being quantities for the former and prices for the latter. In Cournot oligopoly, price is determined by the the sum of the quantities that the firms produce. In oligopoly with perfect complements, the quantity produced is determined by the sum of the prices of the perfect complements. Then, if the demand function and the inverse demand function are the same, quantities in Cournot's oligopoly model lead to precisely the same prices in oligopoly with perfect complements model and vice versa. Thus the two revenue functions write in exactly the same way in terms of the strategies of the players, which is the sense in which Sonnenschein (1968) meant that the models are mathematically equivalent. However, as we shall demonstrate, if one includes the cost structure of the firms in the model, the said equivalence breaks down in general.

There is a fairly extensive literature in industrial organization that deals with various facets of Cournot's complementary monopoly theory. This simple model has been applied to a variety of settings and has been used in multiple policy debates in various areas, including corruption in government services (Shleifer and Vishny, 1993), patents and innovation policy (e.g., Shapiro, 2001), merger theory (Gaudet and Salant, 1992), competition policy (e.g., Gilbert and Katz, 2001), among others. In recent years, renewed and sustained attention to this topic has surfaced in the law and economics literature (e.g. Heller, 1998 and Heller and Eisemberg, 1998) as well as in the public choice literature dealing in particular with property rights (e.g., Buchanan and Yoon, 2000).

With remarkably few exceptions, these different studies share two common features, First, not surprisingly, they typically restrict attention to the usual convenient functional form of linear demand and costs, and thus work with close-form solutions. Second, they put in evidence the main result concerning Cournot's second theory, namely that integrating the n different monopoly suppliers into a single decision-making entity would actually improve market performance in a win-win manner (for all concerned, including consumers), despite the fact that the resulting entity would then be what one may refer to as a super-monopoly.[1]

The main objectives of this paper may accordingly be described as follows. The first is to provide a fairly extensive characterization of the general properties of oligopoly with perfect complements, including basic theoretical preliminaries such as existence and uniqueness of equilibrium points.[2] In part, this amounts to a generalization of the many related results that have appeared in separate contexts over a long period of time. In addition, since the present paper is based on the methodology of supermodular games, this exercise also serves to provide a unifying framework for studies on this model.[3] The second objective is to qualify some conclusions about this model that have drawn close parallels between Cournot's two theories. The duality that Sonnenschein (1968) observed is actually valid only in the absence of production costs. Similarly, based on a linear specification with nice closed form solutions, Buchanan and Yoon (2000) also draw close analogies of both a qualitative and a quantitative sort between the standard Cournot oligopoly (reflecting the commons in a way

[1]Interestingly, this fundamental insight was already described quite clearly by Cournot in his pioneering work. Cournot (1838) found that prices are lower and industry profits higher when a multi-product monopoly produces all the goods instead of having n firms producing the goods.

[2]Surprisingly, relative to the standard Cournot model where such studies have a long history (e.g., Novshek, 1985), no general rigorous theoretic analysis of Cournot's second model is available in the microeconomics literature (to the best of our knowledge).

[3]In particular, we invoke basic results and insights that have appeared in the application of these lattice-theoretic tools in industrial organization theory (see Vives, 1990, 1995; Amir, 1996a; and Amir and Lambson, 2000).

to be made precise later) and Cournot's complementary monopoly model (reflecting the anticommons). Here again, it turns out that these rather striking analogies lack robustness in an essential way: By incorporating linear costs of production into the two models, we show that the parallels largely vanish.

In the overall presentation of the results of this paper, we discuss the relationship between oligopoly with perfect complements and Cournot oligopoly. In particular, we point out the divergences that are engendered by the incorporation of the cost structures into the two models. One example of these differences is that while strategic complementarity of output levels in the standard Cournot oligopoly is not possible in the presence of non-trivial costs (Amir, 1996a), the prices charged by the different monopolies in Cournot's complementary monopoly model may well constitute strategic complements to one another, albeit under general but restrictive assumptions on demand and costs.

The rest of the paper is organized as follows. Section 1.2 provides a precise definition of Cournot's second model and the basic existence proofs for both the asymmetric and the symmetric versions of the model. Section 1.3 conducts a comparative statics analysis of market performance as the number n of components of the system varies. Section 1.4 provides a generalization of the usual argument in the literature about welfare and profit-enhancing integration. Finally, Section 1.5 deals with the inclusion of production costs into the Buchanan and Yoon (2000) setting.

1.1.2 Some economic applications

This subsection discusses some of the applications of the model at hand in various areas of microeconomics. We provide only a short summary here, and refer the reader to the studies themselves for further details and discussion.

One common application deals with patents (see Shapiro, 2001 and Lerner and Tirole, 2004). This is clearly an application of oligopoly with perfect complements if we think of a firm or consumer that wants to develop a new product but might infringe on a number of different patents owned by different parties. Then, the developer has to pay for the usage of all of the patents involved. The patents in this scenario are perfect complements and the consumer needs to buy a license for each one of them.

In their study of corruption in government services, Shleifer and Vishny (1993) discuss the common situation where a private developer needs different permits (e.g., from the fire, water and police departments) to open a new business. This scenario fits into the present model if the officials are assumed to be fully corrupt bribe maximizers.

Feinberg and Kamien (2001) analyze the hold-up problem that can arise in an oligopoly with perfect complements when the acquisition of the multiple parts is sequential. For example, if the government wishes to buy land from different owners in order to build a public project, one owner can wait until the other owners have set prices for their land in order to get a higher benefit for her part of land given that it is necessary for the project. Clearly, the small pieces of land owned by different agents are perfect complements here.

Ellet (1966) uses the metaphor of two different owners of two sequential segments of a road where there are no alternative routes or exits. Gardner, Gaston and Masson (2002) bring this analogy to the real world and apply the oligopoly with perfect complements model to analyze how the Rhine river was tolled in 1254[4].

A classical application of oligopoly with perfect complements is the anticommons problem. This one arises when multiple agents have the right to exclude people from consuming the good (anticommon good) that they own. This is modeled as each owner choosing the price for the anticommon good that maximizes her profit, with the consumers having to pay each of the owners their price in order to use the anticommon good. Buchanan and Yoon (2000) use as an example to illustrate this problem a vacant lot that can be used as a parking lot that has a lower capacity than its open demand. We will return to this example in the last section of the paper.

As shown by the previous examples, oligopoly with perfect complements is a model that has been widely invoked in the literature. Nonetheless, there is a gap in terms of a general characterization of its equilibria, which this paper hopes to fill.

1.2 The Model and some basic results

This section lays out the basic model of Cournot's complementary monopoly and provides some basic existence and uniqueness results. The general asymmetric case and the symmetric case are considered separately. The reason for this is that, when insisting on minimal structural assumptions on the model, the existence ar-

[4]During the period 800-1800, 79 different locations served as toll stations along the Rhine river. The rights to collect tolls were granted by the Emperor, who decided the number, location and amount charged at the toll stations.

guments are quite different across the two cases. In addition, beyond the issues of existence and uniqueness, it is convenient to restrict attention to the symmetric case. This is a key simplifying assumption of the analysis, which is further discussed later on.

Recall that an important auxiliary purpose of this paper is to address and partly correct a widespread but imprecise perception in the literature that the two models that were put forward by Cournot himself in 1938 are duals of one another in some fundamental ways. As the basic results relating to this model are derived, we shall provide a brief comparison with the corresponding results for standard Cournot oligopoly, and assess the similarities and the differences. To avoid confusion between the two models, we shall for the most part refer to the present model as oligopoly with perfect complements (instead of the historical and most commonly used name of Cournot's complementary monopoly). Therefore, we reserve the name of "standard Cournot oligopoly model" for Cournot's much more widely used model of quantity competition.

1.2.1 The Asymmetric Case

Consider an n-firm oligopoly with perfect complements, i.e., a market situation where each of n producers sells one different good as a monopolist, and these goods are totally useless unless purchased together in a fixed ratio to form a final product. W.l.o.g., we assume that this ratio is one-to-one, since we can always appropriately re-normalize the quantities. In other words, consumers wish to purchase a single system with demand function $D(\cdot)$, composed of n different components, each of which is produced and sold by a separate firm acting as a monopoly supplier for that component. This situation is modeled by letting each of the n firms set the price of its own good/component and each consumer buy one unit of each of the n goods, paying the sum of all the prices set by the firms.

This oligopoly with perfect complements is described by (n, K, D, C_i), where n is the number of firms (or goods), K is the maximum price than can be charged for any of the goods in the complementary market[5], $D : [0, \infty) \to [0, \infty)$ is the demand function and $C_i : [0, \infty) \to [0, \infty)$ is firm i's cost function.

Denote the price that the firm under consideration charges by x and the sum of the prices of the remaining $(n-1)$ firms by y. Let $z = x+y$ represent the total price that a consumer has to pay in order to obtain the system (of all the complementary goods).

Firm i chooses the price $x \in [0, K]$ that maximizes its profit given by

$$\pi_i(x, y) = xD(x + y) - C_i[D(x + y)]. \tag{1.1}$$

Its reaction correspondence is

$$r_i(y) = \arg\max\{xD(x + y) - C_i[D(x + y)] : 0 \le x \le K\}. \tag{1.2}$$

Alternatively, we can think of the same firm as choosing $z \in [y, y + K]$ given y, in this case, it maximizes its profit given by

$$\tilde{\pi}_i(z, y) = (z - y)D(z) - C_i[D(z)]. \tag{1.3}$$

[5]The magnitude of K does not play any role in the proofs, so this is assumption is just for convenience.

Define

$$z_i^*(y) = \arg\max\{(z-y)D(z) - C_i[D(z)] : y \le z \le y+K\}. \tag{1.4}$$

Let $\Delta_i(z, y)$ denote the cross-partial derivative of $\tilde{\pi}_i$ with respect to z and y, then

$$\Delta_i(z, y) = -D'(z), \tag{1.5}$$

which turns out to be the same for all firms, so we can suppress the index i in equation 1.5.

Throughout the paper, we maintain the following standard assumptions.[6]

(A1) $D(\cdot)$ is continuously differentiable and $D'(\cdot) < 0$, and

(A2) $C_i(\cdot)$ is twice continuously differentiable and $C_i'(\cdot) \ge 0$.

Under assumption (A1), $\Delta(z, y) > 0$ on the lattice

$$\varphi \hat{=} \{(z, y) : 0 \le y \le (n-1)K, y \le z \le y+K\}.$$

All the proofs are collected in Section 1.7. The following elementary but key result follows directly from the fact that the profit function $\tilde{\pi}_i$ in (1.3) satisfies (a strong notion) of increasing differences on the lattice φ, since $\Delta > 0$ (under smoothness assumptions).

Lemma 1.1. *Assume that the standard assumptions (A1) and (A2) hold. Then, for each n and i, every selection of $r_i(\cdot)$ satisfies the slope condition $\frac{r_i(y')-r_i(y)}{y'-y} > -1$ for all $y' \neq y$.*

[6]Due to the use of supermodularity techniques, the smoothness properties of the demand and cost functions are not necessary for most of the results of this paper. Nevertheless, smoothness is assumed for convenience and ease of interpretation.

Thus, when a firm's rivals all together raise their total price by some amount, the firm may respond by raising or lowering its own price, but in the latter case never by so much that total price ends up going down (relative to the starting point). This property will play a central role throughout the paper.

The central question under consideration in this section is the characterization of respective sufficient conditions on primitives that turn this oligopoly model into a game of strategic substitutes or strategic complements. As will become clear shortly, this issue naturally subsumes the key issue of existence of a pure-strategy Nash equilibrium (henceforth, PSNE) for this model. This is also true in case the game is submodular since it clearly has the aggregation property (defined by the fact that each payoff depends only on own action and on the sum of all other players' actions).[7]

As the model at hand may be viewed as a special case of Bertrand competition with differentiated products, one would expect the prototypical case to satisfy the strategic substitutes property since the goods are complements in demand.[8] The first result indicates that this expectation is essentially correct in that it is fulfilled

[7]A pure-strategy Nash equilibrium for this model might be referred to in a number of different ways, either as a Cournot equilibrium since the concept goes all the way back to the early book by Cournot (1838) or as a Bertrand equilibrium since it deals with a form of price competition. Nonetheless, to avoid a potential for confusion, we shall retain the neutral name of PSNE.

[8]In Singh and Vives (1984), where the case of Bertrand competition with linear demand for differentiated products is considered in some detail, the properties of strategic substitutes and strategic complements coincide exactly with the properties of the goods being complements or substitutes in demand, respectively. However, for non-linear demands, this is no longer true.

under quite a broad scope in terms of the restrictions needed on demand and costs, as captured by the following assumptions on demand and costs.

Theorem 1.2. *Assume that the standard assumptions (A1) and (A2) hold. Then, for each* $n \in N$*, if* $D(\cdot)$ *is log-concave and* $C_i(\cdot)$ *is convex for all* i*, the oligopoly with perfect complements is a game of strategic substitutes and there exists a unique PSNE.*

The conditions of Theorem 1.2 are general enough as to capture most reasonable specifications of Cournot's complementary monopoly in applied settings, including the widely used case of linear demand and costs (see below for such an example). It follows that one can consider this case to represent the prototypical situation for this model.

Despite the fact that the game at hand is a special case of Bertrand competition with complementary products, it turns out that the scope for strategic complementarity of this game is clearly non-trivial, as evidenced by the following (non-degenerate) sufficient conditions.

Theorem 1.3. *Assume that the standard assumptions (A1) and (A2) hold. Then, for each* $n \in N$*, if* $D(\cdot)$ *is log-convex and* $C_i(\cdot)$ *is concave for all* i*, the oligopoly with perfect complements is a game of strategic complements and there exists a (not necessarily unique) PSNE.*

Log-convexity of demand is a rather restrictive condition. Of the commonly used examples, only hyperbolic demand $D(z) = 1/z^{\alpha}$ with $\alpha > 0$ is log-convex.

The limit case of a log-convex demand function is the exponential demand, given by $D(z) = e^{-z}, z \geq 0$, which is strictly convex, log-linear, thus (weakly) log-concave and log-convex.

Observing that the assumptions in the previous results are all in their weak form (as opposed to their strict form), it follows as a direct corollary of the two Theorems that if demand is exponential (i.e., $D(z) = e^{-z}$, $z \geq 0$) and the cost function is linear, then the resulting game must be of both strategic substitutes and of strategic complements. In other words, the reaction curves of all players must be constant functions. We report this formal Corollary in the form of an example.

Example 1. Consider an oligopoly with perfect complements with n firms/goods and a demand function $D(z) = e^{-z}$, $z \geq 0$. Suppose that firm i faces a linear cost function $C_i(q) = c_i q \geq 0$ for producing any output $q \geq 0$. It is easy to derive the reaction curve of firm i (when rivals' total price is $y \geq 0$) as

$$r_i(y) = c_i + 1 \text{ for any } y \geq 0.$$

In other words, each firm has a dominant strategy to price with a mark up of 1 (independent of the actions of the firm's rivals), thus leading to a unique PSNE price vector $(c_1 + 1, c_2 + 1, ..., c_n + 1)$. Consumers pay the total price of $n + \sum_{i=1}^{n} c_i$ and each firm has equilibrium profit equal to $e^{-(n+\sum_{i=1}^{n} c_i)}$. As mentioned, this example serves as an illustration of Theorems 1.2-1.3 as well as Lemma 1.1.

With the general conditions for the existence of PSNE in hand, this ends our consideration of the general case. Henceforth, we shall consider the symmetric case (with identical firms). In particular, this will allow us to conduct comparative statics on the effects of exogenously changing the number of firms based on lattice programming methods, with the number of firms being the relevant parameter.

Comparing these existence results to those for standard Cournot oligopoly, many similarities exist, but also one major difference. The latter model can enjoy strategic complementarities in a global sense only in the abscence of (non-trivial) costs of production. In other words, while a similar duality as the one reflected in the above results holds for the revenue function of Cournot firms, it does not quite extend to the entire profit function. One consequence of this is that, when facing an exponential inverse demand, Cournot firms have dominant strategies if and only if there are no variable costs in production. For more details on these points, see Amir (1996a).

1.2.2 The Symmetric Case

Since each firm faces one and the same demand function for its (firm-specific) good or component, to make all firms identical entails only the standard requirement for a symmetric oligopoly that the firms have the same cost function for the production of their respective goods, denoted then by $C : [0, \infty) \to [0, \infty)$.[9] However, since these goods are not homogeneous in any way, the meaning of identical cost functions is quite different from the standard one (say for Cournot oligopoly). It typically does not entail access to the same technology, but rather that the different goods

[9]For ease of notation, whenever we refer to any of the variables or equations defining this model for now on, we will drop the (firm) index i.

or components cost the same to produce for the same number of units (given the postulated one-to-one composition ratio).

The next Theorem establishes that the standard assumptions alone are sufficient to guarantee the existence of at least one symmetric PSNE for the symmetric oligopoly with perfect complements, and no asymmetric PSNEs.

Theorem 1.4. *Assume that the standard assumptions (A1) and (A2) hold. Then, for each $n \in N$, the oligopoly with perfect complements has at least one symmetric equilibrium and no asymmetric equilibria.*

When all firms are identical, the property captured in Lemma 1.1, that a firm's reaction curve has all of its slopes bounded below by -1, is alone sufficient to yield existence of a (necessarily symmetric) PSNE. This has no counterpart in the asymmetric version of the model.

Recall that a similar property holds in symmetric Cournot oligopoly in terms of output adjustment following a change in rivals' total output, but not universally so. Indeed, in Amir and Lambson (2000), the corresponding property holds only when production enjoys either decreasing returns to scale or a limited form of scale economies.

1.3 On the effects of varying the number of components

A proper study of this oligopoly model requires a good understanding of the effects that added or reduced competition would have on equilibrium prices, per-firm output and profit. Although we are asking how changes in the number of firms

n affect these equilibrium variables, as in Amir and Lambson (2000), the meaning of the question is somewhat different here. Instead of simple entry or exit by one firm, the issue here is a comparison between the two situations where the exact same final product that consumers want can be produced with either n or $(n + 1)$ components, with each firm's cost function for a component being the same in both cases. Depending on the precise context, the actual economic interpretation of this exercise may in fact reflect quite different scenarios. For instance, in the context of a group of patents, it could be that one of the component patents expires (thus implying a move from n to $n - 1$ patents) or that a new patent is added to the group (a move from n to $n + 1$ patents).[10]

While this specific question (involving intermediate values of n) has not really been addressed in the literature on Cournot's complementary monopoly, the comparison between monopoly and the n-firm oligopoly is frequently assessed in specific formulations of this model (we shall have more on this below). The answers provided here correspond to what one would expect, on the basis of the specific formulations analyzed so far, in particular one with linear demand and costs.

As uniqueness of PSNE need not prevail for this model, we denote the equilibrium set for each variable by its corresponding capital letter indexed by n. So with

[10]In the corrupt officials story of Shleifer and Vishny (1993), this might correspond to the government requiring one extra permit (from a new official, say for hygiene) in addition to the existing list of permits. In tolling the Rhine river, it could be that (for a variety of reasons) one of the owning entities decides to offer passage through its own segment toll-free (this then corresponds to a decrease of n by one). Finally, it may be that a policy maker can choose between two technology standards, one involving n components and the other $(n + 1)$ components (with each component produced by a monopolist with the same cost function).

n firms, the equilibrium sets are X_n for per-firm price, Y_n for the firm's $(n-1)$ rivals' cumulative price, Z_n for total price, Q_n for per-firm output, and Π_n for per-firm output.

We say that an equilibrium set for a specific variable in the model is increasing or decreasing in n, when the maximal and minimal points of the set are increasing or decreasing in n, respectively.[11] These are represented by an upper and a lower bar on the relevant variable, respectively.

Theorem 1.5. *Under standard assumptions (A1) and (A2), for each $n \in N$,*

(a) The equilibrium total price Z_n is increasing in n; hence equilibrium per-firm output Q_n is decreasing in n.

(b) The equilibrium per-firm profit Π_n is decreasing in n.

In oligopoly with perfect complements, the addition of one component to the system (or final product) that perfectly complements the existing ones always increases the equilibrium total price. This is very intuitive since now, there is an additional good that the consumer has to buy in order to enjoy all of them. Also not surprisingly, the equilibrium profits of each of the existing firms decreases (though the new monopolist increases from no profit to the same profit as all the others).

The results in Theorem 1.5 have been pointed out before in many particular economic applications, often using particular functional forms. For instance, using the ubiquitous linear demand and zero costs (see Section 1.5 below), Gardner, Gaston

[11] This is a well-known feature of comparative statics conclusions based on supermodularity methods (see e.g., Milgrom and Roberts, 1990, 1994, and Echenique, 2002).

and Masson (2002) show that if the number of segments in road tolling increases, the total price of the tolls goes up while the use of the road and the individual profits of the tolls fall. Moreover, they find that the aggregate profits of the tolls also fall. (The latter result is proved in full generality later on in this paper by Theorem 1.7.)

Shleifer and Vishny (1993) assert that when there is completely free entry of corrupt officials asking for a bribe to provide a service or good produced by the government, the total bribe approaches infinity, driving the provision of the good and the bribe revenues towards zero.

We now investigate the direction of change of the equilibrium per-firm price X_n. As expected, it can take either direction of change depending on the slope of the reaction curve with respect to rivals' cumulative price. Theorem 1.6 gives sufficient conditions for these directions of change. Notice that, as can be seen from the proofs, it is an immediate consequence of Theorems 1.2-1.4 and Lemma 1.10 from Section 1.7, which states that the equilibrium cumulative price of the rest of the $(n-1)$ firms set is increasing in n.

Theorem 1.6. *Assume that the standard assumptions (A1) and (A2) hold. Then, for each* $n \in N$,

(a) If $D(\cdot)$ *is log-convex and* $C(\cdot)$ *is concave, the equilibrium per-firm price* X_n *is increasing in* n.

(b) If $D(\cdot)$ *is log-concave and* $C(\cdot)$ *is convex, the (unique) equilibrium per-firm price* x_n *is decreasing in* n.

As reported earlier, the prototypical case for oligopoly with perfect complements is characterized by strategic substitutes, so that per-firm price will have a more pronounced tendency in general to decrease with the number of components.

Now we turn to study what happens to the equilibrium consumer surplus, total profit and social welfare sets when there is an exogenous change in the number of components. By Theorem 1.5 part (a), the equilibrium total price increases with the number of firms and the equilibrium quantity goes down. Thus, the equilibrium consumer surplus set decreases with more firms in the market. Recall that by Theorem 1.5 part (b), the equilibrium individual profit decreases with the number of products or firms. The following result tells us the stronger result that the equilibrium total profit goes down as well. Combining these results, we conclude that equilibrium social welfare is decreasing in n.

Theorem 1.7. *Assume that the standard assumptions (A1) and (A2) hold, then, for each* $n \in N$,

(a) Equilibrium consumer surplus CS_n *is decreasing in* n.

(b) Equilibrium total profit $n\Pi_n$ *is decreasing in* n.

(c) Equilibrium social welfare W_n *is decreasing in* n.

In the literature on intellectual property rights, many experts have raised the concern that innovation will have a tendency to be stifled in many high-tech industrial sectors by the increasing number of patents. For biomedical research, see Heller and Eisemberg (1998) for more on this. In fact, various calls for a major overhaul of the patenting system are being made both in the U.S. and in Europe.

In some real world examples that fit the setting of oligopoly with perfect complements, the effects captured in this section can lead to dramatically negative consequences for commerce. Shleifer and Vishny (1993) report that, in Zaire, widespread corruption increases the costs of transportation by land so much (due to the large amount of bribes that have to be paid along the way) that it is cheaper to bring the same goods from Europe by ship. In a different but related matter, the excessive number of tolls along the Seine in France around 1400 made shipping costs often more expensive than the goods being transported themselves. In contrast, England was toll free, which is often advanced as a key reason it became the center of commerce (Heilbroner, 1962).

Finally, we extend this analysis to the equilibrium price-cost margin, m_n, defined by $m_n \triangleq x_n - C'[D(z_n)]$. In the empirical literature on market power, this is most often taken as a measure of the level of competition in an industry.

For the model at hand, it turns out that it may increase or decrease with the number of firms depending on whether the demand function is log-convex or log-concave.

Theorem 1.8. *Suppose that the standard assumptions (A1) and (A2) hold. Then, for any $n \in N$, the equilibrium price-cost margin M_n is decreasing in n if $D(\cdot)$ is log-concave but increasing in n if $D(\cdot)$ is log-convex.*

From a comparison between the results in this section and those in Amir and Lambson (2000), it is clear that Cournot oligopoly and oligopoly with perfect complements are not mathematically equivalent theories when the firms are symmetric and the production is costly.

Section 1.5 provides an explicit illustration of this fact.

1.4 Multi-product monopoly as the integrated solution

In the literature, the main focus is on the comparison between n-firm oligopoly with perfect complements and the corresponding integrated solution wherein one multi-product monopolist offers the entire system (of the same n components) at one overall price. Using the same notation as before, the objective function of this n-product monopolist, who faces an n-fold cost of producing the same amount of each component, is

$$\Pi(x) = xD(x) - nC[D(x)] \tag{1.6}$$

It is important to observe that the concept of n-product monopolist is different from special case $n = 1$ in the situation considered in the previous section, i.e., where the entire system amounts to a single component. In other words, the n-product monopolist of this section is not obtained when $n = 1$ is substituted in the model of the previous section (indeed, the objective of the latter would then be $\max_x\{xD(x) - C[D(x)]\}$ instead of (1.6).

The following result compares the market performances of the n-firm oligopoly with perfect complements and of the multi-product monopolist (or the integrated solution).

Proposition 1.9. *Relative to the n-firm oligopoly with perfect complements, the multi-product monopolist solution leads to*

(a) higher total profits,

(b) a lower total price (and thus higher consumer surplus), and

(c) higher social welfare.

This result is fully intuitive and has repeatedly been reported in different settings, using specific functional forms. With linear demand, see e.g., Buchanan and Yoon (2000) and Gardner, Gaston and Masson (2002).

1.5 Multi-product monopoly versus Oligopoly

This section considers the simple framework of Buchanan and Yoon (2000) where Cournot's original two models are compared in a variety of ways under linear demand and costless production. The main purpose here it to establish that the findings in Buchanan and Yoon (2000), which these authors invoked to claim a striking symmetry between the commons and the anticommons, do not carry over to the case of costly production.[12]

1.5.1 A summary of Buchanan and Yoon (2000)

Buchanan and Yoon (2000) (hereafter, BY, 2000) consider a vacant lot that can be used as a (capacity-constrained) parking. If the vacant lot is a common good, it will be used more than efficiently but if it is privatized, the new $n > 1$ owners will sell permits that the potential users have to buy in order to park in the lot. Any person who wants to park in the vacant lot has to buy one permit from each one of

[12]Costless production is a reasonable assumption in the context of the scenario analysed by Buchanan and Yoon (2000), as well as in some of the other commonly used situations that are captured by Cournot's complementary monopoly model. However, the typical situation will naturally feature production costs.

the owners. The outcome is that the vacant lot will be used less than efficiently. The first case illustrates the commons problem and the second one, the anticommons one.

The commons problem can be seen as a Cournot oligopoly with $n > 1$ firms because the owners of the common good decide how much of it to use in order to maximize their profits. Given that the owners cannot exclude others from the usage of the common good, they maximize their profits given the choice of usage of the other owners. The efficient level of usage of the good is equal to the output that a monopolist would choose in this setting; thus, in this case, the relevant concept of monopoly is given by the standard single-product monopolist.

On the other hand, the anticommons problem fits the setting of an oligopoly with perfect complements with $n > 1$ firms. Firms are equivalent to "excluders" that choose the price of their permits sold to the potential users in order for them to use the anticommon good. In this case, the relevant concept of monopoly that provides the efficient level of permits (and thus, usage) is a multi-product monopolist that sells all the permits as a bundle (with perfect complements).

BY(2000) solve the equilibrium for the four cases of interest under a linear demand and *zero costs*. With inverse demand $P(q) = a - bq$, a single-product monopoly solves $\max_q q(a - bq)$ and each firm in Cournot oligopoly solves $\max_q q(a - b(q + q'))$ where y' is rivals' total output.

With direct demand $D(z) = \frac{a-z}{b}$, the multi-product monopolist solves $\max_x x(\frac{a-x}{b})$ and a firm in the oligopoly with perfect complements solves $\max_x x(\frac{a-(x+y)}{b})$ (with y defined as before).

	BY(2000) (c=0)			Present paper (c>0)		
$n > 1$	Q^*_{BY}	P^*_{BY}	Π^*_{BY}	Q^*	P^*	Π^*
Single-product monopoly	$\frac{a}{2b}$	$\frac{a}{2}$	$\frac{a^2}{4b}$	$\frac{a-c}{2b}$	$\frac{a+c}{2}$	$\frac{(a-c)^2}{4b}$
Cournot oligopoly, n firms	$\frac{na}{b(n+1)}$	$\frac{a}{n+1}$	$\frac{na^2}{b(n+1)^2}$	$\frac{n(a-c)}{b(n+1)}$	$\frac{a+nc}{n+1}$	$\frac{n(a-c)^2}{b(n+1)^2}$
Loss in profit from 1 to n firms			$\frac{a^2(n-1)^2}{4b(n+1)^2}$			$\frac{(a-c)^2(n-1)^2}{4b(n+1)^2}$
Multi-product monopoly, n goods	$\frac{a}{2b}$	$\frac{a}{2}$	$\frac{a^2}{4b}$	$\frac{a-nc}{2b}$	$\frac{a+nc}{2}$	$\frac{(a-nc)^2}{4b}$
Oligopoly with perfect complements, n firms	$\frac{a}{b(n+1)}$	$\frac{na}{n+1}$	$\frac{na^2}{b(n+1)^2}$	$\frac{a-nc}{b(n+1)}$	$\frac{n(a+c)}{n+1}$	$\frac{n(a-nc)^2}{b(n+1)^2}$
Loss in profit from 1 to n firms			$\frac{a^2(n-1)^2}{4b(n+1)^2}$			$\frac{(a-nc)^2(n-1)^2}{4b(n+1)^2}$

Table 1.1: Equilibrium total output (Q^*), equilibrium total price (P^*) and equilibrium industry profit (Π^*) for different settings.

The first three columns of Table 1.1 summarize the results in BY(2000), denoted by the subindex BY. From this table, we observe their main results listed below.

(1) The single product monopolist and the multi-product monopolist produce the same amount of output ($a/[2b]$) and thus, charge the same price ($a/2$) and earn the same profit ($a^2/[4b]$).

(2) Cournot oligopoly produces more than the single-product monopolist ($na/[b(n+1)] > a/[2b]$) and thus, its price is lower ($a/[n+1] < a/2$).

(3) Oligopoly with perfect complements produces less than the multi-product monopoly ($a/[b(n+1)] < a/[2b]$), hence, it charges a higher price ($na/[n+1] > a/2$).

(4) Cournot oligopoly and oligopoly with perfect complements earn the same profit ($na^2/[b(n+1)^2]$).

(5) Each of the two monopolists earn more profit than each oligopolist ($a^2/[4b] > na^2/[b(n+1)^2]$).

(6) The losses in industry profit from changing from single-product monopoly to Cournot oligopoly and from multi-product monopoly to oligopoly with perfect complements are the same ($a^2(n-1)^2/[4b(n+1)^2]$).

Based on the similarity shown in items (1) and (4)-(6), BY(2000) conclude that Cournot oligopoly and oligopoly with perfect complements lead to symmetric tragedies. In particular, these tragedies are reflected in the profit losses of changing from one to n firms having the same magnitude in both models.

1.5.2 Adding production costs to the BY model

In line with the results of the present paper, we now show that the symmetry in equilibrium industry profits easily breaks down with the inclusion of costs of production. We consider linear costs of production given by $C(q) = cq$ with $c > 0$.

Because the multi-product monopolist produces n goods that are different, it needs n separated plants to produce them and since our setting is symmetric, it pays n times the cost of producing the optimal amount of bundles. Then, the optimization problem of the multi-product monopoly becomes choosing the price x of the bundle that maximizes its profit given by $(x - nc)(\frac{a-x}{b})$. The single-product monopolist, a firm competing à la Cournot and a firm in the oligopoly with perfect complements chooses x that maximizes its profit given by $x(a - bx - c)$, $x(a - b(x + y') - c)$ and $(x - c)(\frac{a-(x+y)}{b})$, respectively, where y' and y are defined as earlier.

The solutions and equilibria to the maximization problems and games listed above are summarized in the last three columns of Table 1.1 (with $a - nc > 0$). With $c = 0$, we recover the results of BY(2000). When $c > 0$, we have

(7) The single-product monopoly earns more profits than the Cournot oligopoly ($(a - c)^2/[4b] > n(a - c)^2/[b(n + 1)^2]$).

(8) The multi-product monopoly earns more profits than the oligopoly with perfect complements ($(a - nc)^2/[4b] > n(a - nc)^2/[b(n + 1)^2]$).

(9) The single-product monopoly earns more profits that the multi-product monopoly ($(a - c)^2/[4b] > (a - nc)^2/[4b]$).

(10) Cournot oligopoly earns more industry profits than the oligopoly with perfect

complements $(n(a-c)^2/[b(n+1)^2] > n(a-nc)^2/[b(n+1)^2])$.

(11) The loss in industry profit of changing from single-product monopoly to Cournot oligopoly is bigger than the loss in industry profit of having an oligopoly with perfect complements instead of a multi-product monopoly $((a-c)^2(n-1)^2/[4b(n+1)^2] > (a-nc)^2(n-1)^2/[4b(n+1)^2])$.

Thus, we have illustrated that the effects on industry profits of adding $(n-1)$ firms to the single-product monopolist are in general, different from the effects on industry profits when we have an oligopoly with perfect complements instead of a multi-product monopoly.

The idea that the tragedies of the commons and anticommons are not symmetric is discussed by Vanneste *et. al.* (2006). Using two experiments, a lab experiment versus a scenario experiment, they conclude that the behaviors of the players facing the same versions of a commons dilemma and an anticommons dilemma are different. In particular, they find that the players act more aggressively (with higher decision variables) when they face the anticommons dilemma. Although this might be explained through the specification of the demand function, this paper brings up the concern that the commons and anticommons problems are not symmetric in general.

1.6 Conclusions

The results in this paper give conditions for the existence of equilibrium in the more general setting of an oligopoly with perfect complements, i.e., when the firms face different costs of production. In the symmetric case, when all the firms face the same cost of production function, at least one symmetric equilibrium exists under our standard assumptions (A1) and (A2). The reason is that $\Delta(\cdot, \cdot)$ defined in equation (1.5) is always strictly greater than zero. Moreover, the paper characterizes the symmetric equilibrium, i.e., it looks into the effects on the equilibrium variables of interest when the number of goods in the market exogenously changes.

It is important to be careful when interpreting the comparative statics results. Here, by entry of a firm, we mean the addition of a new good that perfectly complements the existing ones in the market. In fact, the results in Theorem 1.5 do not allow us to do comparative statics with respect to the number of firms/products when at least one firm is producing more than one good. These results hold only if one firm is producing exactly one good out of the n goods that are the perfect complements in the market.

In addition, the paper enriches previous discussions about the similarities between Cournot oligopoly and oligopoly with perfect complements, showing that they are different theories when the costs of production are non negligible. In general, the characterization of the equilibrium and the comparative statics change, but these are not the only differences that we find, as can be seen from the application in Section 1.5.

An extension of this model is to allow firms to produce more than one good or merging, i.e., firms getting together to produce two or more goods and sell them separately or in a bundle.

In conclusion, this paper characterizes the equilibria of an oligopoly with perfect complements, starting from the asymmetric case, where the firms have different cost structure. For that case, we give conditions such than an equilibrium exists. In the symmetric case, where the firms have the same cost structure, a symmetric equilibrium always exists and based on lattice-theoretic methods, we are able to do comparative statics on these symmetric equilibria when the number of firms/goods in the market changes.

1.7 Proofs

We begin by defining a key mapping for every $n \in N$, which can be thought of as a normalized cumulative best-response correspondence. This mapping is analogous to the one used by Amir and Lambson (2000) and is useful in dealing with symmetric equilibria in the present context too.

$$B_n : [0, (n-1)K] \longrightarrow 2^{[0,(n-1)K]}$$

where

$$B_n(y) = \frac{n-1}{n}(x' + y). \tag{1.7}$$

Here, x' represents the firm's best-response, i.e., the price that maximizes its profit in (1.1) given the cumulative price y for the remaining $(n-1)$ firms. If $x' \in [0, K]$ and $y \in [0, (n-1)K]$, then the (set-valued) range of B_n is as given. Also, a fixed point of B_n, $\hat{y}$, clearly yields a symmetric PSNE where $\hat{x}' = \hat{y}/(n-1)$, i.e., each of the responding firms will set the same price as the other $(n-1)$ firms.

Proof of Lemma 1.1.

Under (A1), the cross partial derivative of the maximand in (1.3), $\Delta(z, y)$, is strictly positive on the lattice

$$\varphi = \{(z, y) : 0 \leq y \leq (n-1)K, y \leq z \leq y + K\}.$$

The feasible set $[y, y + K]$ is ascending in y. Then, by a strengthening of the basic monotonicity theorem of Topkis (1978) due to Amir (1996c) and Edlin and Shannon (1998), every selection of z_i^* is strictly increasing in y as long as it is interior.

Since $z_i^*(y) = r_i(y) + y$, it follows directly that $r_i(\cdot)$ has the given slope property.□

Proof of Theorem 1.2.

By a dual argument to Theorem 1.3 (proved below), the profit $\pi_i(x, y)$ exhibits the dual single-crossing property under the hypotheses of the Theorem. Since the game at hand is an aggregative game of strategic substitutes, by Tarski (1955) and Novshek (1985), an equilibrium exists.

To show that there is a unique PSNE, use Lemma 1.1 and the first part of this proof to conclude that all the slopes (of all the selections) of $r_i(\cdot)$ lie in $(-1, 0]$. Then, by a well known (contraction-like) argument, the equilibrium is unique (see e.g., Amir, 1996b). □

Proof of Theorem 1.3. Milgrom and Shannon (1994) prove, for a Bertrand duopoly with differentiated and complementary products, that if each demand function $\widetilde{D}_i(x, y)$

is log-supermodular and the cost function is concave, then $\pi_i(x, y)$ satisfies the single-crossing property in (x, y). Since log-supermodularity of $\widetilde{D}_i(x, y)$ translates into the log-convexity of $D(x + y)$ in our setting, the proof of this Theorem follows as a special case of their result. Hence, the game is a game of strategic complements and by Tarski's fixed-point Theorem (Tarski, 1955), an equilibrium exists. □

Proof of Theorem 1.4.

By the proof of Lemma 1.1, every selection of z^* is increasing in y. Recall that x' denotes the firm's best-response to y, thus $z^*(y) = x' + y$. This implies that for every $n \in N$, every selection of B_n as defined by equation (1.7) is increasing in y. Then, by Tarski's fixed-point Theorem, (any selection of) B_n has a fixed-point that implies the existence of a symmetric equilibrium of the oligopoly with perfect complements.

Next, we prove that no asymmetric equilibrium can exist. By the proof of Lemma 1.1, every selection of z^* is strictly increasing. This means that for each $z' \in z^*$ corresponds at most one y such that $z' = x' + y$ (z' is the best-response to y); then, for each total equilibrium price z', each firm must charge the same price $x' = z' - y$, with $y = (n - 1)x'$, i.e., no asymmetric equilibrium exists. □

Before proceeding with the rest of the proofs, we need the following intermediate Lemmas.

Lemma 1.10. *Assume that the standard assumptions (A1) and (A2) hold. Then, for every number of firms* $n \in N$*, the equilibrium cumulative prices of* $(n - 1)$ *firms* *set, Y_n, is increasing in n.*

Proof of Lemma 1.10.

By Topkis's Theorem, the maximal and minimal selections of B_n, denoted by $\overline{B}_n$ and $\underline{B}_n$ respectively, exist. Furthermore, the largest equilibrium cumulative price for $(n-1)$ firms, $\overline{y}_n$, is the largest fixed-point of $\overline{B}_n$. $\overline{B}_n(y)$ is increasing in n for every fixed y. Hence, by Theorem A.4 in Amir and Lambson (2000), the largest fixed-point of $\overline{B}_n$, $\overline{y}_n$, is also increasing in n. Using an analogous argument with $\underline{B}_n$ shows that the the smallest equilibrium cumulative price for $(n-1)$ firms, $\underline{y}_n$, is increasing in n. □

Lemma 1.11. *Assume that the standard assumptions (A1) and (A2) hold. Then, for every number of firms $n \in N$, $\overline{\pi}_n = \pi(\underline{x}_n, (n-1)\underline{x}_n) \geq \underline{\pi}_n = \pi(\overline{x}_n, (n-1)\overline{x}_n)$.*

Proof of Lemma 1.11.

We prove that $\overline{\pi}_n = \pi(\underline{x}_n, (n-1)\underline{x}_n)$, a similar argument shows that $\underline{\pi}_n = \pi(\overline{x}_n, (n-1)\overline{x}_n)$. To this aim, observe that $\tilde{\pi}(z, y)$ is decreasing in y, then, $\overline{\tilde{\pi}}_n = \tilde{\pi}(\underline{z}_n, \frac{(n-1)}{n}\underline{z}_n) = \pi(\underline{x}_n, (n-1)\underline{x}_n)$. Now, we show that $\overline{\pi}_n = \pi(\underline{x}_n, (n-1)\underline{x}_n)$. Suppose not, then it exists $\tilde{x}_n \in X_n$ such that $\pi(\tilde{x}_n, (n-1)\tilde{x}_n) > \pi(\underline{x}_n, (n-1)\underline{x}_n)$, then, $\pi(\tilde{x}_n, (n-1)\tilde{x}_n) = \tilde{\pi}(\tilde{z}_n, \frac{(n-1)}{n}\tilde{z}_n) > \pi(\underline{x}_n, (n-1)\underline{x}_n) = \overline{\tilde{\pi}}_n$, where $\tilde{z}_n = n\tilde{x}_n$, which contradicts the fact that $\overline{\tilde{\pi}}_n$ is the maximal element in the set $\tilde{\Pi}_n$, thus, $\pi(\underline{x}_n, (n-1)\underline{x}_n)$ is the maximal per-firm profit equilibrium. □

Proof of Theorem 1.5.

(a) From the proof of Lemma 1.1, we know that every selection of z^* is increasing in

y. Since $\overline{y}_n$ is increasing in n (Lemma 1.10), we conclude that $\overline{z}_n$ is increasing in n. Using an analogous argument, $\underline{z}_n$ is increasing in n.

(b) This follows as a direct corollary of the proof of *Theorem 1.7(b)* where the stronger result $n\overline{\pi}_n \geq (n+1)\overline{\pi}_{n+1}$ is proved.$\square$

Proof of Theorem 1.6.

(a) By the proof of Theorem 1.3, the extremal selections from $r(\cdot)$ are increasing in y. Then, $\overline{x}_n = \bar{r}(\overline{y}_n)$, and given that $\overline{y}_n$ is increasing in n (by Lemma 1.10), so is $\overline{x}_n$. A similar argument follows for $\underline{x}_n$.

(b) By Theorem 1.2, a unique equilibrium exists which is symmetric by Theorem 1.4. Also, by the proof of Theorem 1.2 we know that every selection of $r(\cdot)$ is decreasing in y. Since in equilibrium y_n is increasing in n (by Lemma 1.10) and $x_n = r(y_n)$, x_n is decreasing in n. $\square$

Proof of Theorem 1.7.

(a) The consumer surplus, $CS(\cdot)$, at any total price z and for any n is given by

$$CS(z) = \int_z^\infty D(t)\ dt,$$

which is decreasing in z.

Then,

$\overline{CS}_n - \overline{CS}_{n+1} = \int_{\underline{z}_n}^\infty D(t)\ dt - \int_{\underline{z}_{n+1}}^\infty D(t)\ dt = \int_{\underline{z}_n}^{\underline{z}_{n+1}} D(t)\ dt \geq 0.$

The inequality follows by Theorem 1.5 part (a), $\underline{z}_{n+1} \geq \underline{z}_n$.

A similar argument using $\overline{z}_n$ and $\overline{z}_{n+1}$ proves that $\underline{CS}_n$ is decreasing in n.

(b) We prove that $n\overline{\pi}_n \geq (n+1)\overline{\pi}_{n+1}$. The result that $n\underline{\pi}_n$ is decreasing in n follows by a similar argument using $\overline{x}_n$. Consider the following relations:

$$\begin{aligned}
\overline{\pi}_n &= \underline{x}_n D(n\underline{x}_n) - C[D(n\underline{x}_n)] \\
&\geq [(n+1)\underline{x}_{n+1} - (n-1)\underline{x}_n] D[(n+1)\underline{x}_{n+1} - (n-1)\underline{x}_n + (n-1)\underline{x}_n] \\
&\quad - C[D[(n+1)\underline{x}_{n+1} - (n-1)\underline{x}_n + (n-1)\underline{x}_n]] \\
&\geq \left[(n+1)\underline{x}_{n+1} - \frac{(n-1)(n+1)}{n}\underline{x}_{n+1}\right] D[(n+1)\underline{x}_{n+1}] - C[D[(n+1)\underline{x}_{n+1}] \\
&= \frac{(n+1)}{n}\underline{x}_{n+1} D[(n+1)\underline{x}_{n+1}] - C[D[(n+1)\underline{x}_{n+1}] \\
&= \frac{(n+1)}{n}\left[\underline{x}_{n+1} D[(n+1)\underline{x}_{n+1}] - \frac{n}{n+1} C[D[(n+1)\underline{x}_{n+1}]\right] \\
&\geq \frac{(n+1)}{n}\left[\underline{x}_{n+1} D[(n+1)\underline{x}_{n+1}] - C[D[(n+1)\underline{x}_{n+1}]\right] \\
&= \frac{(n+1)}{n}\overline{\pi}_{n+1}.
\end{aligned}$$

The first equality follows by Lemma 1.11 and the first inequality by the PSNE property. The deviation $(n+1)\underline{x}_{n+1} - (n-1)\underline{x}_n$ from the equilibrium price is positive since it is equal to $\underline{z}_{n+1} - \underline{z}_n + \underline{x}_n$, and by Theorem 1.5 part (a), $\underline{z}_{n+1} \geq \underline{z}_n$. The second inequality follows also by Theorem 1.5(a), in particular using the fact that $\frac{(n+1)\underline{x}_{n+1}}{n} \geq \underline{x}_n$ (from $\underline{z}_{n+1} = (n+1)\underline{x}_{n+1} \geq \underline{z}_n = n\underline{x}_n$). The last inequality is due to $\frac{n}{n+1} < 1$.

(c) It is clear that $\overline{W}_n = \overline{CS}_n + n\overline{\pi}_n$. Then

$\overline{W}_n - \overline{W}_{n+1} = [\overline{CS}_n - \overline{CS}_{n+1}] + [n\overline{\pi}_n - (n+1)\overline{\pi}_{n+1}] \geq 0.$

The inequality follows because both terms on the RHS of the equality are positive by parts (a) and (b). A similar argument proves that $\underline{W}_n$ is decreasing in n. □

Proof of Theorem 1.8.

Let us consider (say) the maximal point of the equilibrium price-cost margin set.

Since $D(\cdot)$ is log-concave (log-convex), we have $\frac{D'(z)}{D(z)} \geq (\leq) \frac{D'(z')}{D(z')}$ for all $z' > z$. Thus,

$$-\frac{D(z')}{D'(z')} + \frac{D(z)}{D'(z)} \leq (\geq) 0. \tag{1.8}$$

Now, the first-order condition for the oligopoly with perfect complements can be written as

$$D(z_n) + m_n D'(z_n) = 0,$$

which implies that

$$m_n = -\frac{D(z_n)}{D'(z_n)}.$$

If $D(\cdot)$ is log-concave (log-convex), m_n is decreasing (increasing) in z_n, then, $\overline{m}_n = -\frac{D(\underline{z}_n)}{D'(\underline{z}_n)}$ $(\overline{m}_n = -\frac{D(\overline{z}_n)}{D'(\overline{z}_n)})$.

Thus, $\overline{m}_{n+1} - \overline{m}_n = -\frac{D(\underline{z}_{n+1})}{D'(\underline{z}_{n+1})} + \frac{D(\underline{z}_n)}{D'(\underline{z}_n)}$ $\left(\overline{m}_{n+1} - \overline{m}_n = -\frac{D(\overline{z}_{n+1})}{D'(\overline{z}_{n+1})} + \frac{D(\overline{z}_n)}{D'(\overline{z}_n)}\right)$, which is negative (positive) if $D(\cdot)$ is log-concave (log-convex), by equation 1.8 and Theorem 1.5 part (a), $\underline{z}_{n+1} \geq \underline{z}_n$ $(\overline{z}_{n+1} \geq \overline{z}_n)$. □

Proof of Proposition 1.9.

(a) This follows from a rather standard argument. Since the n-product monopolist can replicate whatever price vector the oligopoly can use, it is obvious that the former can achieve a higher total profit than the latter.

(b) The sum (across the n firms) of the first order conditions at a symmetric PSNE is (where z stands for total price)

$$nD(z) + zD'(z) - nC'[D(z)]D'(z) = 0. \tag{1.9}$$

From equation (1.6), the first order condition for the n-product monopolist's solution is (where z stands for total price)

$$D(z) + zD'(z) - nC'[D(z)]D'(z) = 0. \tag{1.10}$$

Since for any $n > 1$, the LHS of (1.9) is an upward shift of the LHS of (1.10), the extremal solutions (which are the extremal zeros of the LHS's) of (1.9) are higher than those of (1.10).

Hence, price is lower for the n-product monopolist. It follows that consumer surplus is higher with the n-product monopolist.

(c) This follows directly from (a) and (b).□

CHAPTER 2
ON OLIGOPOLY WITH POSITIVE NETWORK EFFECTS AND INCOMPATIBLE NETWORKS

2.1 Introduction

It is very common to find industries with positive network effects, i.e., industries where the willingness to pay of the consumers increase with the number of people that are purchasing the good. A widely used example in the literature is telecommunications such as telephone and fax. A consumer will be more interested in buying a cell phone if more people acquire a compatible cell phone, among other reasons, the consumer will be able to communicate with more persons and in the case of smart phones, the consumer expects more software to be developed if the good is more popular.

In their seminal work about network effects, Katz and Shapiro (1985) identify three kind of oligopolies with network effects; in all the cases, the firms produce goods that are substitutes and compete in quantity. The first model is when all the goods produced by the firms are completely compatible; in the second one, any two goods produced by two different firms are incompatible, and finally, there is the partial compatibility model, where there are groups of goods that are compatible among them but incompatible with the goods outside of that group.

In this paper, we center our attention to the study of oligopolies with positive network effects and complete incompatibility. An example of this industry is video games (Church and Gandal, 1992). Several firms in the market produce different video game consoles that are not compatible among them, in the sense that a game that is designed for a particular console cannot be played using a different one. Then, every firm possesses their own network and the consumers benefit from the size of it, which is known in the literature as demand-side economies of scale. If more consumers purchase a particular console, they expect to have a bigger variety of games because the software developers might have more incentives to create more if the number of users increase, also, the agents benefit with a bigger size of the network because they will have more people to play with.

Other common examples of industries with incompatible networks come from diverse technologies in the 1980's and 1990's. For instance, personal computers, when IBM and Macintosh were not compatible; video cassette recorders, that will be discussed with more detail later on, and digital music systems such as digital compact cassette and mini disc (see Church and Gandal, 1992; Cusumano, Mylonadis and Rosenbloom, 1992 and Katz and Shapiro, 1986).

Specifically, this paper focuses in the study of the symmetric model, i.e., when the firms face the same demand and same costs of production. One of the objectives of this study is to find minimal conditions such that an equilibrium exists in these kind of symmetric industries. The concept of equilibrium that we use is the one introduced by Katz and Shapiro (1985), the fulfilled expectations Cournot equilibrium, where all the firms maximize their profits and the consumers' expectations are fulfilled, i.e., the output of each firm equals the expected size of the consumers.

An interesting feature of this symmetric model is that besides symmetric equilibria, asymmetric equilibria in all their possibilities can easily arise, as first acknowledged by Katz and Shapiro (1985). First, it can be the case that some firms stay out of the market by producing an output of zero while the firms producing a positive output produce the same output among them, i.e., a symmetric equilibrium with less than all the firms active. This equilibrium includes the natural monopoly equilibrium, where only one firm is active producing a strictly positive output. The other possibility is when at least two firms produce a strictly positive but different amount of output between them.

These asymmetric equilibria arise when some firms are more successful than others with their products because the consumers expect so. An illustration of this fact follows from the competition between Betamax and Video Home System (VHS) in the 1970's-1980's. Betamax and VHS were two different and incompatible formats of home video cassette recorders (VCRs). Not only the sizes of the cassettes for the VCRs were different, also the tape-handling mechanisms and the technology to read the tapes differed, which kept the two formats completely incompatible. Betamax was introduced in 1975 by the Sony Corporation and VHS, just one year after by the Victor Company of Japan (JVC). Although the sales of the Betamax kept increasing until 1985, its market share was below the VHS by 1978. Finally, Sony and the firms that adopted this format stopped producing the Betamax (see Cusumano, Mylonadis and Rosenbloom (1992) for more details). Although in this market more than one firm was producing the same format, this case helps us to illustrate the emergence of asymmetric equilibria in the presence of firms that could be considered symmetric given the similarities between the two technologies. At the beginning of the 1980s, VHS had a higher market share, and by the end of the decade, Betamax was out of the market.

An important question in this industry is viability, i.e., whether a network will succeed or not. This is a particularly important issue in our model since it allows for pure networks good, i.e., for goods whose value for the consumers is zero if the expected size of the network is zero, independently of the total production. Since pure network goods are included in the model, it is possible that all or a subset of the networks do not emerge, thus, besides existence of equilibria, we look for conditions such that non trivial -symmetric or asymmetric- equilibria exist.

In their seminal work, Katz and Shapiro (1985) do not consider the viability problem, since their separable demand function does not allow for pure network goods and trivial equilibria are not a problem. In a later work, Katz and Shapiro (1986) discuss the viability problem for a dynamic setting, where there are two periods and success of a network mainly depends on whether it is sponsored or not. Other characteristics that dictate the viability of a network are the differences in the technology and second-mover advantages.

Church and Gandal (1992) study the viability problem from a different perspective. They look at viability as a consequence of the complementary goods suppliers decisions. In other words, we can think of the firms in the industry producing hardware, for example video game consoles, that require software to be enjoyed, video games; in this case, the software is the complement to the hardware. Church and Gandal (1992) research the success of the hardware producers given the software suppliers decisions, i.e., which hardware they decide to support (which network they decide to join). In contrast to previous works on viability, the present paper looks on conditions on the demand and costs structure to predict whether all the networks succeed or only some of them, for a single period.

On a separate topic, we look at the effects on the extremal equilibria of adding firms to the industry, including effects on the equilibrium consumer surplus, industry profits and social welfare.

Finally, we compare the equilibria for the oligopolies with complete compatibility and complete incompatibility. Several authors like Katz and Shapiro (1986), Economides and Flyer (1997) and Chen, Doraszelski and Harrington (2009) have addressed this question which is closely related to viability. Typically, to study the standardization problem (the incentives of a firm to adhere to an existing network or to differentiate itself), the game is modeled in two stages. In the first one, the firms choose the technology, and in the second one, they compete in price or quantity. Chen, Doraszelski and Harrington (2009) extend this model to a dynamic stochastic setting. Economides and Flyer (1997) and Chen, Doraszelski and Harrington (2009) discuss the relevance of the strength of the network effects in their results, particulary the former ones.

In this paper, we simplify the standardization problem by giving specific conditions on the demand such that the firms, consumers and society are better off with compatible products. The results are provided for a static game where the firms compete in quantity. We find out that the oligopoly with complete compatibility produces more than the one with incompatible networks. Similarly, the first one generates a higher social welfare. The comparison between consumer surplus and industry profit is not clear, but in line with Economides and Flyer (1997), it strongly depends on the size of the network effects. For instance, if the latter are big enough to offset the market effect, the firms will prefer to have compatible networks because a bigger network benefits them through a higher willingness to pay from the consumers. Since social welfare is higher under compatible networks, we can conclude that at least one group (consumers or firms) is always better off under this setting.

2.2 Oligopoly with positive network effects and complete incompatibility

2.2.1 The model and the assumptions

In this section we describe the model, which is a static game of oligopolistic competition with positive network effects and complete incompatibility. This is, we describe a market situation where the firms produce substitute goods and the consumers' willingness to pay for any good is increasing in the number of agents that purchase the good, also known in the networks literature as demand-side economies of scale. In our model, the goods are substitutes but not compatible among the firms; thus, every firm possesses its own network. This model is a generalization of the oligopoly with network effects and complete incompatibility described by Katz and Shapiro (1985). In equilibrium, every firm maximizes its profit given the production of the rest of the firms and their expected network size, which is fulfilled by their production. This notion of equilibrium is know as fullfilled expectations Cournot equilibrium (FECE) (Katz and Shapiro, 1985), and will be formally defined later on.

According to the description above, in this market structure we have n symmetric firms that produce substitute goods. Every firm in the market faces the inverse demand given by the function $P(z, s)$, where z denotes the total output in the market and s, the expected size of the network. Since the goods produced by the firms are totally incompatible, every firm has its individual network with expected size s, which is not necessarily the same among the firms. If every consumer purchases at most one unit of the good, s accounts for the expected number of agents to consume the good.

We assume that the firms are identical, thus, they all face the same cost of production given by the function $C(\cdot)$. Hence, for every s given, firm i chooses the output that maximizes its profit given by

$$\pi(x, y, s) = xP(x + y, s) - C(x),$$

where x is the firm's output level and y is the joint output of the other $(n - 1)$ firms. Notice that the firm does not choose s, this is exogenous for the firm as well as y, i.e., the firm does not have the power to influence the consumers' expectations about the network size. Then, the firm's best reaction correspondence is given by

$$x(y, s) = argmax\{\pi(x, y, s) : x \geq 0\}.$$

Alternatively, one can think of firm i as choosing total output $z = x + y$ that maximizes

$$\tilde{\pi}(z, y, s) = (z - y)P(z, s) - C(z - y),$$

with best-reaction correspondence

$$z(y, s) = argmax\{\tilde{\pi}(z, y, s) : z \geq y\},$$

given a particular y and s. Then, it should be the case that $z(y, s) = x(y, s) + y$.

As mentioned above, the equilibrium of this game is given by what Katz and Shapiro (1985) called a "fulfilled expectations Cournot equilibrium" (FECE), i.e., the equilibrium is given by a vector of individual outputs $(x_{1n}, x_{2n}, ..., x_{nn})$ and a vector of expected networks sizes $(s_1, s_2, ..., s_n)$ such that:

(1) $x_{in} \in argmax\{xP(x + \sum_{j \neq i} x_{jn}, s_i) - C(x) : x \geq 0\}, and$

(2) $x_{in} = s_i,$

for all $i \in \{1, 2, ..., n\}$. Throughout the paper, the subindex n denotes that the variable in question is under equilibrium, when the subindex i is added, it means that the variable in equilibrium is specific for firm i. Whenever possible in the paper, for example, when the equilibrium is symmetric, the subindex i will be dropped for simplicity in the notation.

The following assumptions will be assumed through all the paper:

(A1) $P : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ is twice continuously differentiable, $P_1(z, s) < 0$ and $P_2(z, s) > 0$.

(A2) $C : [0, \infty) \rightarrow [0, \infty)$ is twice continuously differentiable, increasing and C(0)=0.

(A3) $x_i \leq K$, for each firm i.

(A4) $\Delta_1(z, y, s) \triangleq -P_1(z, s) + C''(z - y) > 0$ on $\varphi \triangleq \{(z, y, s) : z \geq y, y \geq 0, s \geq 0\}$.

(A5) $\Delta_2(z, s) \triangleq P(z, s)P_{12}(z, s) - P_1(z, s)P_2(z, s) > 0$ on φ.

The first two assumptions are standard in the literature, in particular, $P_2(z, s) > 0$ reflects the positive network effects or demand-side economies of scale, i.e., the increment in the willingness to pay of the consumers when more people is expected to buy the good.

The capacity constraint assumption (A3) is needed for technical reasons but the magnitude of K does not affect the results. The fourth assumption guarantees that every selection of the total output best-response correspondence, $z(y, s)$, is increasing in y for all s given. For a detailed discussion, see Amir and Lambson (2000).

The last assumption, (A5), implies that the inverse demand function is strictly log-supermodular in (z,s) which implies that every selection of $z(y, s)$ is increasing in s for every y given. The latter result follows because (A5) guarantees that the alternative profit function $\tilde{\pi}(z, y, s)$ has strict increasing differences in (z,s) and by Topkis (1998). Economically, (A5) tells us that the demand is more elastic when the expected size of the network is bigger, which reflects the demand-side scale economies discussed in the network industries literature. See Amir and Lazzati (2011) for a deeper discussion.

Notice that there is no special restriction on the value of $P(z, 0)$, i.e., it can take values that are greater or equal than zero. This characteristic of the inverse demand function allows the model to account for pure and and mixed network goods. Pure networks good are those that do not have stand-alone value, i.e., $P(z, 0) = 0$, meaning that if the expected size of the good's network is zero, no consumer will value this good. On the other hand, $P(z, 0) > 0$ reflects a mixed network good, where the consumers value the product even if the expected size of the network is zero.

2.2.2 Existence of equilibrium and viability

In their pioneering work on network externalities, Katz and Shapiro (1985) noticed that the symmetric oligopoly modeled in this paper can lead to three different kinds of equilibria: 1) symmetric equilibria where all the firms produce the same output; 2) m-active-firm symmetric equilibria where $m < n$ firms produce a positive output that is the same among them and the rest of the $n - m$ firms produce nothing, and 3) asymmetric equilibria where $m \geq 2$ firms produce positive but different equilibrium outputs.

Networks externalities are the only responsible for the existence of asymmetric equilibria. In a standard Cournot symmetric model, where there are no network effects, asymmetric equilibria may only arise in the presence of strong economies of scale. In other words, asymmetric equilibria in standard Cournot oligopoly model may only arise when its corresponding Δ_1 is negative, which in this paper is ruled out by assumption (A4). Moreover, in standard Cournot model it cannot be the case that two firms have positive but different production (Amir and Lambson, 2000).

The possibility of having asymmetric equilibria is not the only difference that we find between Cournot oligopoly with and without network effects under assumption (A4). As we will see in Section 2.2.3, in the presence of networks externalities, entry of firms may increase equilibrium prices and profits of the firms. According to Amir and Lambson (2000), in Cournot oligopoly with no network effects and under its corresponding (A4), the price and per-firm profits always decrease with the entry of firms.

In this section, we state that under the assumptions (A1)-(A5), a symmetric equilibrium always exist; nonetheless, the question whether asymmetric equilibria exist, in any of the two ways described above, prevails. This paper covers this issue, in particular, sufficient conditions are given such that an m-active-firm symmetric equilibrium exists; these conditions will be discussed later on. All the proofs are shown in Section 2.5 and assumptions (A1)-(A5) are assumed through all the paper.

Theorem 2.1. *For each $n \in N$, The Cournot oligopoly with positive network effects has (at least) one symmetric equilibrium.*

In industries with network effects, it is common to face equilibria where none of the networks emerge, i.e., where in equilibrium, all the firms stay out of the market by producing nothing. The following lemma characterizes this kind of equilibrium, moreover, it can be shown that the existence of the trivial equilibrium in the n-oligopoly implies the existence of the trivial equilibrium for the $(n+1)$-oligopoly with positive network effects and incompatible networks, as stated in Lemma 2.3. The latter result implies that whenever a zero individual output (an thus, zero aggregated output) is an equilibrium for the industry with n firms, an oligopoly with $m > n$ identical firms (among them and among the industry with n firms) will have it. Notice that the converse is also true, the n-firms oligopoly having the trivial equilibrium implies that any identical industry differing only in having a less number of firms, say $m < n$, will have the trivial equilibrium as well.

Lemma 2.2. *The trivial outcome is an equilibrium if and only if* $xP(x, 0) \leq C(x)$ *for all* $x \in [0, K]$.

Lemma 2.3. *The trivial outcome is an equilibrium for* $n \in N$ *firms if and only if the trivial outcome is an equilibrium for* $(n + 1)$ *firms.*

A consequence of Lemma 2.3, is that if the trivial equilibrium is not an equilibrium for an industry with n firms, it will not be an equilibrium for the industry with one more firm and viceversa. In this case, a non-trivial equilibrium will exist for both industries, as a consequence of the existence theorem, Theorem 2.1. This result is formalized later on, in Theorem 2.5.

One natural question to ask is under what conditions of the primitives defining the industry, one can find a non-trivial equilibrium, i.e., an equilibrium where at least one of the networks is successful so that the industry emerges through at least, one firm. Theorem 2.5 gives conditions such that a symmetric equilibrium other than the trivial one exists. It turns out that the conditions that guarantee the existence of a non-trivial symmetric equilibrium for the n-firms oligopoly can be reinterpreted as conditions that lead to a m-active-firm symmetric equilibrium. Before presenting the result, we introduce Lemma 2.4 that leads to part (iii) in Theorem 2.5. In this paper, $x_n(s)$ denotes the individual output symmetric equilibrium correspondence for each of the $i = 1, ..., n$ firms when there are n firms and s is exogenous.

Lemma 2.4. *If* $0 \in x_n(0)$*, then* $x_n(0) = 0$*. If in addition* $P(0,0) = C'(0)$*, then* $x'_n(0)$ *exists, it is also single-valued and right-continuous at* 0*, and*

$$x'_n(0) = -\frac{P_2(0,0)}{(n+1)P_1(0,0) - C''(0)}. \tag{2.1}$$

If $P(0,0) < C'(0)$*, then* $x_n(0) = 0$ *and* $x'_n(0) = 0$*.*

Theorem 2.5. *A non-trivial symmetric equilibrium exists if*

(i) $0 \notin x_n(0)$*, i.e., zero is not an equilibrium individual output (or* $xP(x,0) > C(x)$ *for some* $x \in (0, K]$*);*

A non-trivial symmetric equilibrium with $m \leq n$ *active firms exists if*

(iii) $0 \in x_n(0)$*,* $P(0,0) = C'(0)$ *and* $P_1(0,0) + P_2(0,0) > -mP_1(0,0) + C''(0)$*; or*

(iv) $0 \in x_n(0)$*,* $C''(\cdot) \geq 0$ *and* $P(z,s) + \frac{z}{m}P_1(z,s) \geq C'(z/m)$ *for some* $s \in (0, K]$ *and for all* $z \leq ms$*.*

The result in Theorem 2.5 part (i) is immediate because we know that at least a symmetric equilibrium always exist (Theorem 2.1).

Theorem 2.5 is closely related to the viability conditions for firms with compatible networks in Amir and Lazzati (2011). These results are very powerful since they are analogous for compatible and incompatible networks. Moreover, in the latter case, parts (ii) and (iii) of Theorem 2.5 can be generalized in order to guarantee the existence of a m-active-firm symmetric equilibrium, $m < n$. By Lemma 2.3, if zero is an equilibrium for the oligopoly with n firms, then it will be for the oligopoly with $m < n$ firms. Thus, we find sufficient conditions such that a non-trivial equilibrium exists for the industry with $m < n$ firms; but we need to show that zero is a best-response for the rest of the $n - m$ firms. The intuition is that if zero is a symmetric equilibrium, zero will also be a best response for a firm that face positive production by the rest of the firms. This argument is formalized in the proof and follows immediately by Lemma 2.2 and (A1).

Now that we have discussed the existence of equilibria and more importantly, of non-trivial equilibria, we are ready to do some analysis on the effects of entry. The next section is of interest because the inclusion of demand-side economies of scale in a Cournot model lead to unfeasible results in the standard Cournot model under the same assumptions (in particular, under the corresponding (A4)). For instance, the price might increase with the number of firms, even if the output increases as well. Similarly, due to network effects, the firms might be able to increase their profits with the entry of competitors.

2.2.3 Entry of firms

In this section, we analyze what happens to the symmetric equilibrium when the number of firms exogenously increase in the market, i.e., we look at the changes in the equilibrium variables of interest when the equilibrium is symmetric. The results hold for the maximal and minimal equilibria, we cannot tell the direction of change of all the points in the equilibrium sets when we increase the number of firms since the comparative statics results in this paper are based on the results due to Milgrom and Roberts (1990, 1994), that are valid only for the extremal equilibria of the games.

The results are listed for the maximal equilibria, denoted by the corresponding equilibrium variable (the variable sub-indexed by n) with an upper bar. The reader should keep in mind that these results are true for the minimal equilibria as well, denoted by a lower bar.

The following definitions will be useful in doing the comparative statics analysis.

$\Delta_3(z) = P_1(z, z/n) + [1/n]P_2(z, z/n)$, and

$\Delta_4(x) = P_1^2(nx, x) - [P(nx, x) - C'(x)]P_{11}(nx, x)$.

$I_n = [z_n, z_{n+1}]$.

Notice that $\Delta_3(\cdot)$ denotes the change in the market price with an increase in the aggregate output along the fulfilled expectation path.

The use of the interval I_n relaxes the sufficient conditions, in the sense that they do not have to hold globally but for the interval. As shown by Theorem 2.6, the total output extremal equilibria are increasing in the number of firms, thus, the interval I_n is well defined for the extremal outcomes.

Theorem 2.6. *For any* $n \in N$

(i) $\bar{z}_{n+1} \geq \bar{z}_n$; *and*

(ii) $\bar{P}_{n+1} \leq \bar{P}_n$ *if* $\Delta_3(\cdot) \leq 0$ *on* $\underline{I}_n = [\underline{z}_n, \underline{z}_{n+1}]$.

In this market structure, more firms produce an equal or bigger aggregated output under a symmetric equilibrium, like in the standard Cournot oligopoly game. Nonetheless, the effect in the equilibrium market price is ambiguous given the networks effect. In equilibrium, the aggregated output affects the market price in a direct way and in an indirect way through the network effect. If the network effect is not big enough to offset the direct effect, the price decreases in equilibrium. On the other hand, if the network effect is big enough, the price might go up given that the willingness to pay of the consumers increment is sufficiently large.

Notice that one implication of Theorem 2.6 is that if a non-trivial equilibrium exists for n firms, then a non-trivial equilibrium must exist for $n+1$ firms, since now this industry with more firms produces at least the same output than with a firm less. This conclusion is highly related to the implications of Lemma 2.3 by taking its negation, the difference is that the result does not follow directly by the non existence of the trivial equilibrium, but by the existence of a non-trivial equilibrium.

The following proposition is a generalization of Katz and Shapiro's (1985) result. It follows from Theorems 2.1 and 2.6 and assumption (A1). Existence is given by Theorem 2.1 with an increasing aggregated output in the number of firms by Theorem 2.6. The clue is that if $(n-m)$ firms best-respond by producing nothing when they face a m-active-firm symmetric equilibrium, then zero is as well a best-response when $(m+1)$ firms play a symmetric equilibrium.

Proposition 2.7. *Let $m < n$. If an m-active-firm symmetric equilibrium exists, then an $(m+1)$-active-firm symmetric equilibrium exists with aggregated equilibrium output greater than that for the m-active-firm symmetric equilibrium.*

An interesting consequence of Proposition 2.7 is that if a monopolistic equilibrium exists for n firms, i.e., when the industry emerges and one firm produces the monopolistic output keeping the rest of the firms outside of the market, then a non-trivial symmetric equilibrium exists for the n firms with bigger equilibrium industry output. Additionally, when the monopolistic outcome is an equilibrium, non-trivial equilibria with only m firms active, $1 < m < n$, will exist.

The following Theorem gives sufficient conditions for the direction of change of the individual outputs. Notice that the results are stated for the case where the extremal equilibria are interior. If they are not interior, i.e., the firms produce zero or K in the presence of n firms, the result becomes trivial.

Theorem 2.8. *At an interior equilibrium, per-firm outputs are such that*

(i) $\bar{x}_{n+1} \geq \bar{x}_n$ *if* $\Delta_4(\cdot) \leq 0$*; and*

(ii) $\bar{x}_{n+1} \leq \bar{x}_n$ *if* $\Delta_4(\cdot) \geq 0$.

Based on the previous theorem, we can analyze what happens to the profit of the firms when there is exogenous entry. As opposed to the standard Cournot oligopoly model, it is possible for the firms to increase their profits with more competition as a consequence of the networks effects. If the latter is sufficiently large, the price may increase enough to make the firms better off. The profits are evaluated at the maximal equilibria.

Theorem 2.9. *For any* $n \in N$*, per-firm profits are such that*

(i) $\pi_{n+1} \geq \pi_n$ *if* $\bar{x}_{n+1} \geq \bar{x}_n$ *and* $P(\bar{z}_{n+1}, \bar{x}_{n+1}) \geq P(\bar{z}_n, \bar{x}_n)$*;*

(ii) $\pi_{n+1} \leq \pi_n$ *if* $\bar{x}_{n+1} \leq \bar{x}_n$.

Notice that the conditions $\bar{x}_{n+1} \geq \bar{x}_n$ and $\bar{x}_{n+1} \leq \bar{x}_n$ can be replaced by $\Delta_4(\cdot) \leq 0$ and $\Delta_4(\cdot) \geq 0$ respectively, by Lemma 2.8. Nonetheless, the second pair of conditions are more restrictive since they are sufficient conditions for $\bar{x}_{n+1} \geq \bar{x}_n$

and $\bar{x}_{n+1} \leq \bar{x}_n$ respectively, thus, we keep the conditions of the theorem in the most relaxed way.

Now we turn to analyze how equilibrium consumer surplus, industry profit and welfare change with exogenous entry of firms. Consumer surplus with total output z and expected size of each network s is defined by $CS(z, s) = \int_0^z P(t, s)dt - zP(z, s)$. Social welfare when industry output is z, expected size of every network is s and all the firms produce the same amount of output is given by $W(z, s) = \int_0^z P(t, s)dt - nC(z/n)$.

Proposition 2.10. *At the highest equilibrium output, consumer surplus is increasing in the number of firms, $CS_{n+1} \geq CS_n$, if at least one of the following assumptions hold*

(i) $P_{n+1} \leq P_n$;

(ii) $\bar{x}_{n+1} \geq \bar{x}_n$ and $P_{12}(z, s) \leq 0$ for all z, s, or

(iii) $\bar{x}_{n+1} \leq \bar{x}_n$ and $P_{12}(z, s) \geq 0$ for all z, s.

It is straightforward that condition (i) gives us the result given that total output goes up (by Theorem 2.6 part (i)) and price goes down. As can be seen from the proof, a more general condition than (ii) and (iii) is that $P(t, \bar{x}_{n+1}) - P(t, \bar{x}_n) \geq P(\bar{z}_n, \bar{x}_{n+1}) - P(\bar{z}_n, \bar{x}_n)$ for all $t \in [0, z_n]$, but conditions (i) and (ii) are sufficient for this inequality to hold.

The following proposition gives conditions such that in equilibrium, the industry profit increases with the number of firms. The first result follows directly from Theorem 2.9 part (i). The second part is very intuitive, since the price increases more than the average cost with a new competitor, the firms earn a higher profit. The function $A(\cdot)$ denotes the average cost function, i.e., $A(x) = C(x)/x$.

Proposition 2.11. *At the highest equilibrium output, industry profit is increasing in the number of firms, $(n+1)\pi_{n+1} \geq n\pi_n$, if one of the following assumptions hold*

(i) $\bar{x}_{n+1} \geq \bar{x}_n$ and $P(\bar{z}_{n+1}, \bar{x}_{n+1}) \geq P(\bar{z}_n, \bar{x}_n)$; or

(ii) $P(\bar{z}_{n+1}, \bar{x}_{n+1}) - P(\bar{z}_n, \bar{x}_n) \geq A(\bar{x}_{n+1}) - A(\bar{x}_n)$.

Industry profit is decreasing in the number of firms, $(n+1)\pi_{n+1} \leq n\pi_n$, if

(iii) $\bar{x}_{n+1} \leq \bar{x}_n$ and $A(\frac{n+1}{n}\bar{x}_{n+1}) \leq A(\bar{x}_{n+1})$.

Finally, we look at the changes in social welfare when new firms enter the market.

Proposition 2.12. *At the highest equilibrium output, welfare is increasing in the number of firms, $W_{n+1} \geq W_n$, if one of the following assumptions hold*

(i) $P(z, \bar{x}_{n+1}) - P(z, \bar{x}_n) \geq A(\bar{x}_{n+1}) - A(\bar{x}_n)$ for all z, or

(ii) $\bar{x}_{n+1} \geq \bar{x}_n$.

2.3 Complete compatibility versus complete incompatibility

In this section, we compare equilibrium variables of interest between the oligopolies with positive network effects under complete compatibility and complete incompatibility. In particular, we look at the highest equilibrium individual outputs and their corresponding market prices, profits, consumer surpluses and social welfare; the results hold as well for the lowest equilibria.

An oligopoly with positive network effects and complete compatibility is defined in a similar way as the one with complete incompatibility, described in this paper, except that in the first model, the goods produced by the firms are perfectly compatible with each other, thus, there is only one network for the goods with expected size S. Hence, each one of the firms in the industry maximize their profit given by

$$\pi(x, y, S) = xP(x + y, S) - C(x),$$

where the primitives and variables of the previous equation are described as in Section 2.2.1.

It is crucial that the lector keeps in minds that although the profit functions look identical for either model, they are fundamentally different by the interpretation put on the parameters s and S. In oligopoly with incompatible networks, s denotes the expected size of the firm's own network[1]; in oligopoly with complete compatibility, S denotes the size of the whole network, which in equilibrium matches the industry production. Amir and Lazzati (2011) provide a throughout characterization of the latter model.

Before proceeding with the results, we clarify the notation and introduce a definition and two Lemmas that are useful in proving the main results. For the n-firm equilibrium notation, we still use the variable sub-indexed by the number of firms in the industry but now we add a super-index that denotes complete compatibility (C) or complete incompatibility (I).

[1] One can think about it as being s_i for firm i, but we have dropped the sub-index along this paper for simplicity in the notation.

On the other hand, we define $\Delta_5(\cdot)$ by

$\Delta_5(z) = P_1(z,z) + P_2(z,z)$,

which denotes the change of the market price when aggregated output changes in an industry with complete compatibility and along the equilibrium path.

Lemmas 2.13 and 2.14 deal with the comparison between individual output equilibria when one of them is a corner solution, i.e., zero or K.

Lemma 2.13. $x_n^C = 0$ *if and only if* $x_n^I = 0$.

Lemma 2.14. *If* $x_n^I = K$*, then,* $x_n^C = K$.

Notice that the reverse of Lemma 2.14 is in general not true, this is, every firm producing their maximum capacity, K, being an equilibrium for the oligopoly with complete compatibility does not necessarily imply that it will be an equilibrium for the oligopoly with complete incompatibility. Hence, in this hypothetical case, a firm under complete incompatibility would produce K units of output or less. It turns out that this is not the only case where in equilibrium, a firm under complete incompatibility produces the same or less than a firm under complete compatibility; this is a general result for the extremal equilibria of the models, as stated in the following Theorem.

Theorem 2.15. *At the highest equilibrium output and for every* $n \in N$

(i) $\bar{x}_n^C \geq \bar{x}_n^I$*, and*

(ii) $P_n^C \geq P_n^I$ *if* $\Delta_5(\cdot) \geq 0$ *on* $[n\bar{x}_n^I, n\bar{x}_n^C]$.

It is straightforward that Theorem 2.15 part (i) implies that an industry with complete compatibility will produce at last the same aggregated output than the industry with complete incompatibility in equilibrium (just multiply by n the inequality in part (i)). This is a generalization of the result by Katz and Shapiro (1985) who prove the same result for a linear and separable inverse demand function with costless production.

Theorem 2.15 part (ii) says that if the network effect is sufficiently big to offset the competitive effect, reflected by $\Delta_5(\cdot) \geq 0$, the consumers have a higher willingness to pay if the goods are compatible, since the size of the network is bigger in this case. This results in a higher equilibrium price for the compatible networks. Theorem 2.15 implies that a firm prefers complete compatibility when $\Delta_5(\cdot) \geq 0$; since the firms can charge a higher price and the production is higher, the firms get a higher profit. This is consistent with Economides and Flyer (1997), who argues that the welfare of a firm depends on the strength of the network effect.

The previous result is summarized in the following theorem, together with the analysis of consumer surplus and social welfare. Notice that society is always better off under complete compatibility, by Theorem 2.16 part (iii), but we cannot conclude in general that the firms or the consumers always prefer one kind of industry. Nevertheless, we can conclude that complete compatibility always benefits at least one group, consumers or firms.

Theorem 2.16. *At the highest equilibrium outputs and for every* $n \in N$

(i) $\pi_n^C \geq \pi_n^I$ *if* $\Delta_5(\cdot) \geq 0$;

(ii) $CS_n^C \geq CS_n^I$ *if* $P_n^I \geq P_n^C$ *or* $P_{12}(z, s) \leq 0$, *and*

(iii) $W_n^C \geq W_n^I$.

Theorem 2.16 part (ii) relates to Church and Gandal (1992) in the sense that consumer surplus cannot be compared in a global sense. Which setting gives the consumers a higher surplus relies on the characteristics of the market, specifically, of the demand. To see that consumer surplus can be bigger under complete incompatibility than under complete compatibility at the highest equilibrium individual output, let us revisit Example 3 in Amir and Lazzati (2011).

Example. Let $P(z,s) = max\{a - z/s^3, 0\}$ denote the inverse demand function of an industry with n symmetric firms that face zero production costs. Assume that $a \geq n/K^2$ and $K > 1$. As shown by Amir and Lazzati (2011), the reaction function of a firm is given by

$$x(y,s) = \begin{cases} max\{(as^3 - y)/2, 0\}, & \text{if } (as^3 - y)/2 < K, \\ K & \text{if } (as^3 - y)/2 \geq K. \end{cases}$$

If the networks are completely compatible, the FECE industry output set is given by $z_n^C = \{0, \sqrt{(n+1)/na}, nK\}$. Then, for the highest equilibrium $\overline{z}_n^C = nK$, the consumer surplus is $CS_n^C = 1/(2nK)$.

Now, if the networks are completely incompatible, the FECE industry output set is given by $z_n^I = \{0, n\sqrt{(n+1)/a}, nK\}$. Then, $CS_n^I = n^2/(2K)$ for the highest FECE $\overline{z}_n^I = nK$. Thus, in this setting, FECE consumer surplus is higher under complete incompatibility than under complete compatibility.

Notice that $P_n^C = a - 1/(nK)^2$ is greater than $P_n^I = a - n/K^2$ and $P_{12}(z,s) = 3/s^4 > 0$, i.e., the sufficient conditions of Theorem 2.16 part (ii) are violated. A simple argument generalizes this result stated in the next Corollary, if K is a symmetric FECE for the incompatible oligopoly and $P_{12}(z,s) \geq 0$, then $CS_n^I \geq CS_n^C$.

Finally, we have that $\pi_n^C = K(a - 1/(nK)^2) \geq K(a - n/K^2) = \pi_n^I$ and $W_n^C = anK - 1/(2nK) \geq W_n^I = anK - n^2/(2K)$, as Theorem 2.16 parts (i) and (iii) predicts given that $\Delta_5(z) = P_1(z,z) + P_2(z,z) = 2/z^3 > 0$.

From the example above, it is apparent that a sufficient condition for the consumers to prefer incompatible networks is that the incompatible firms produce at their maximum capacity when the inverse demand function exhibits increasing differences in (z,s). By Lemma 2.14, this implies that the compatible firms also produce at their maximum capacity. As a more general result, it can be shown that if both industries produce exactly the same output (including asymmetric equilibria in the incompatible setting), the consumers will prefer incompatibility. The reason is that the market output is the same, a smaller network decrease the price and by supermodularity of the inverse demand function, the competitive effect is reinforced. The previous result is summarized in the following corollary.

Corollary 2.17. *At the highest equilibrium outputs and for every $n \in N$, if $z_n^I = z_n^C$ and $P_{12}(z,s) \geq 0$, then, $CS_n^I \geq CS_n^C$.*

2.4 Conclusions

This paper generalizes the theory in Katz and Shapiro (1985) on oligopolies with positive network effects and complete incompatibility. In particular, it allows for pure network goods, where the viability question becomes relevant since now, trivial equilibria where none of the firms produce is a possibility. A symmetric setting of this model, where all the firms face the same demand and costs of production, becomes particularly interesting because asymmetric equilibria may arise in any shape: it could be that a subset of the firms is active and produce an equal amount of goods among them or that at least two firms produce a positive and different level of output. Thus, the viability question is covered for both the symmetric equilibrium with all the firms active and with a subset of them active.

A challenge in the literature regarding this model has been to characterize completely asymmetric equilibria, when two or more firms produce a positive and different output. Future research may look towards this direction. This topic is particularly relevant because it may explain the success of some networks over their rivals, for example, the triumph of VHS over Beta in the late 80's, despite their high level of substitution and Beta being the precursor of VCRs and the leader in the market by the middle 70's.

The present paper also provides some interesting public policy implications. It is shown that in a general setting with symmetric firms, social welfare is higher when the networks are compatible; similarly, there is a higher production. The former result implies that at least the consumers or the firms are better off with complete compatibility, but it is not certain that both groups simultaneously are. Sufficient conditions such that the firms and the consumers prefer compatibility over incompatibility are provided.

Intuitively, when the networks effect is high enough that it offsets the demand effect, a higher network generated by the complete compatibility of the firms' networks allows the industry to charge a sufficiently high price to obtain higher profits than under complete incompatibility (since we already know that compatibility increases production). On the contrary, if the firms charge a higher price under incompatibility, the consumers will prefer the cheaper product in the compatible setting with a higher output, that clearly leads to a higher consumer surplus. This is only one sufficient condition that provides the consumers with a higher surplus under compatibility; in fact, this paper presents an example were the consumers are better off with complete incompatibility.

Summarizing, this paper provides a general framework for symmetric oligopolies with positive network effects and complete incompatibility. It gives minimal conditions for the existence of equilibrium with particular interest in non-trivial symmetric and asymmetric equilibria. Similarly, it studies the effects of more competition in the market and compares the equilibrium outcomes of this model with that when the networks are totally compatible.

2.5 Proofs

First, we introduce the following lemmas that will be useful in the main proofs of this paper.

Lemma 2.18. $\tilde{\pi}(z, y, s)$ *has the strict single-crossing property in* (z, s).

Proof of Lemma 2.18. The proof follows by Amir and Lazzati (2011), Lemma 12. □

Lemma 2.19. *$\pi(x, y, s)$ has the strict single-crossing property in (x, s), thus, every selection of the best-response correspondence $x(y, s)$ increases in s.*

Proof of Lemma 2.19. Notice that $\pi(x, y, s) = \tilde{\pi}(x + y, y, s)$. By Lemma 2.18, for any $z > z'$ and $s > s'$, $\tilde{\pi}(z, y, s') \geq \tilde{\pi}(z', y, s') \Rightarrow \tilde{\pi}(z, y, s) > \tilde{\pi}(z', y, s)$. Let $x > x'$, then $x+y > x'+y$ and $\tilde{\pi}(x+y, y, s') \geq \tilde{\pi}(x'+y, y, s') \Rightarrow \tilde{\pi}(x+y, y, s) > \tilde{\pi}(x'+y, y, s)$, thus, $\pi(x, y, s') \geq \pi(x', y, s') \Rightarrow \pi(x, y, s) > \pi(x', y, s)$, which proves the first part of the lemma. Notice that the feasible correspondence $[0, K]$ is ascending in s, then, by Topkis's Theorem (1998), every selection of $x(y, s)$ is increasing in s. □

Lemma 2.20. *Every selection of the best-response correspondence $z(y, s)$ increases in both y and s.*

Proof of Lemma 2.20. The proof follows by Amir and Lazzati (2011), Lemma 1. □

Proof of Theorem 2.1. Let the expected size of the network be s for each firm. By Amir and Lambson (2000), a symmetric Cournot equilibrium exists and comes from a fixed point of the correspondence

$$B_s : [0, (n-1)K] \to 2^{[0,(n-1)K]}$$

$$y \to \frac{n-1}{n} z(y, s).$$

Now we need to prove that $x_n(s)$, the set of symmetric Cournot equilibrium individual outputs for each firm $i = 1, ..., n$, has fixed points. By Lemma 2.20, every selection of $z(y,s)$ is increasing in s. Then, by Milgrom and Roberts (1990), the maximal and minimal fixed points of $B_s(y)$, $\bar{y}(s)$ and $\underline{y}(s)$ respectively, increase in s. Hence, by symmetry and by Lemma 2.20, the extremal selections of the correspondence $x_n : [0,K] \longrightarrow [0,K]$, $\bar{x}_n(s) = \bar{y}(s)/(n-1)$ and $\underline{x}_n(s) = \underline{y}(s)/(n-1)$, are increasing in s. Thus, by Tarski's fixed point theorem (1955), $\bar{x}_n(s)$ and $\underline{x}_n(s)$ have fixed points that can be seen as symmetric FECE's. □

Proof of Lemma 2.2. By definition, an individual (and hence, industry) output of 0 is a symmetric FECE if and only if $0 \in x_n(0)$, i.e., $0 \in x(0,0)$. This holds if and only if $\pi(0,0,0) \geq \pi(x,0,0)$ $\forall x \in [0,K]$, i.e., $0 \geq xP(x,0) - C(x)$ $\forall x \in [0,K]$. □

Proof of Lemma 2.3. By proof of Lemma 2.2, $0 \in x_n(0)$ if and only if $\pi(0,0,0) \geq \pi(x,0,0)$ $\forall x \in [0,K]$ if and only if $0 \in x_{n+1}(0)$. □

Lemma 2.21. *Let $\tilde{x}_n(s)$ be an increasing selection of $x_n(s)$. Then $\tilde{x}_n(s)$ is differentiable for almost all s, and, if $\tilde{x}_n(s) \in (0,K)$ for $s > 0$, its slope is given by ($\tilde{x}_n$ stands for $\tilde{x}_n(s)$)*

$$\frac{\partial \tilde{x}_n(s)}{\partial s} = -\frac{\tilde{x}_n P_{12}(n\tilde{x}_n, s) + P_2(n\tilde{x}_n, s)}{(n+1)P_1(n\tilde{x}_n, s) + n\tilde{x}_n P_{11}(n\tilde{x}_n, s) - C''(\tilde{x}_n)}. \quad (2.2)$$

Proof of Lemma 2.21. If $\tilde{x}_n(s)$ is interior, it satisfies the first order condition

$$\tilde{x}_n(s)P_1(n\tilde{x}_n(s), s) + P(n\tilde{x}_n(s), s) - C'(\tilde{x}_n(s)) = 0. \quad (2.3)$$

Since $\tilde{x}_n(s)$ is increasing, it is differentiable almost everywhere w.r.t. Lebesgue measure. Hence, differentiating both sides of equation (2.3) with respect to s on a subset of full Lebesgue measure and reordering terms gives us the result. □

Proof of Lemma 2.4. To see that $x_n(0)$ is single-valued and equal to zero, notice that $0 \in x_n(0)$ imply, by Lemma 2.2, that $xP(x,0) \leq C(x)$ for all $x \in [0,K]$. Therefore, by (A1), $xP(x+y,0) < C(x)$ for all $x, y > 0$. This tell us that 0 is a strictly dominant strategy with s=0 and thus, $x_n(0) = 0$.

Now suppose that $P(0,0) = C'(0)$, i.e., zero is an interior equilibrium, and that $\tilde{x}_n(s)$ is an increasing selection of $x_n(s)$. Take a sequence $s_k \downarrow 0$ such that a selection $\tilde{x}_n(s)$ is differentiable at s_k for all k, this is possible because $\tilde{x}_n(s)$ is an increasing function, moreover, $lim_{k\to\infty}\tilde{x}_n(s_k)$ exists. By Fudenberg and Tirole (1991), $x_n(s)$ is upper hemi-continuous, therefore, $lim_{k\to\infty}\tilde{x}_n(s_k) \in x_n(0) = \{0\}$, i.e., $lim_{k\to\infty}\tilde{x}_n(s_k) = 0$.

Now, by (A1), (A2) and $lim_{k\to\infty}\tilde{x}_n(s_k) = 0$, the right-hand side of equation (2.2) is right-continuous in s at 0. Doing $s = s_k$ and taking limit as $k \to \infty$ in the right-hand side of (2.2), it follows that $lim_{k\to\infty}\frac{\partial\tilde{x}_n(s_k)}{\partial s}$ exists and it is given by the right-hand side of equation (2.1). Because this argument is valid for all sequences (s_k) taken from the subset of full Lebesgue measure of the domain $[0,K]$, $\partial\tilde{x}_n(s)/\partial s \mid_{s=0}$ exists, is continuous at 0 and given by (2.1).

Notice that the previous argument holds for the extremal selections of $x_n(s)$, $\bar{x}_n(s)$ and $\underline{x}_n(s)$, since they are increasing in s by the proof of Theorem 2.1. Thus, their derivatives at $s = 0$ are equal and given by the right-hand side of equation 2.1, moreover, we have that $max\{\partial x_n(s)/\partial s\mid_{s=0}\} = \partial\bar{x}_n(s)/\partial s\mid_{s=0} = \partial\underline{x}_n(s)/\partial s\mid_{s=0} = min\{\partial x_n(s)/\partial s\mid_{s=0}\}$, then, $\partial x_n(s)/\partial s\mid_{s=0}$ exists, is single-valued, continuous and given by the right-hand side of equation (2.1).

Finally, assume that $P(0,0) < C'(0)$. Then, by (A1), $P(0,s) < C'(0)$ for s sufficiently small and in consequence, $x_n(s) = 0$ for all such s. Hence, $x_n'(0) = 0$. □

Let $\Pi(z,s) \triangleq \frac{n-1}{n}\left[\int_0^z P(t,s)dt - nC(z/n)\right] + \frac{1}{n}[zP(z,s) - nC(z/n)]$, a weighted average of welfare and industry profits when s is the same for all firms. Thus, we have the following result.

Lemma 2.22. *Suppose that $C(\cdot)$ is convex. If $z^* \in argmax\{\Pi(z,s), 0 \le z \le nK\}$, then, $x^* \equiv \frac{z^*}{n} \in x_n(s)$, for all $n \in N$ and $s \in [0,K]$.*

Proof of Lemma 2.22. By Amir and Lazzati (2011), Lemma 14, for any $n \in N$ and $s \in [0,K]$, if $z^* \in argmax\{\Pi(z,s), 0 \le z \le nK\}$, then, $z^* \in z_n(s)$, where $z_n(s)$ is the total output equilibrium correspondence for a given s. Then, by symmetry, $z^* \in z_n(s)$ implies that $x^* \equiv \frac{z^*}{n} \in x_n(s)$. □

Proof of Theorem 2.5.

(i) If the trivial outcome is not part of the equilibrium set, Theorem 2.1 guarantees there is a symmetric FECE with strictly positive individual output.

(ii) Parts (ii) and (iii) use the following argument; for generality in the result, assume for now that we have $m \le n$ firms. By the proof of Theorem 2.1, the maximal selection of $x_m(s)$, $\bar{x}_m(s)$, is increasing in s. Suppose that there exists $s' \in (0, K]$ such that $\bar{x}_m(s') \geq s'$. The facts that $\bar{x}_m(\cdot)$ is increasing and $\bar{x}_m(s') \geq s'$ (by assumption) imply that for all $s \in [s', K]$, $\bar{x}_m(s) \in [s', K]$. Then, $\bar{x}_m(s)$ is an increasing function that maps $[s', K]$ into itself. By Tarski's fixed point theorem (1955), there exists $s'' \in [s', K]$ such that $\bar{x}_m(s'') = s''$, which is a strictly positive FECE. Therefore, we only need to show that there exists $s \in (0, K]$ such that $\bar{x}_m(s) \geq s$ to prove that there is a non-trivial symmetric equilibrium for $m \leq n$ firms. To this end, it suffices to show that $x'_m(0) > 1$ which implies that there is a small $\epsilon > 0$ for which $\bar{x}_m(\epsilon) > \epsilon$. By hypothesis, $0 \in x_n(0)$ which implies by Lemma 2.3 that $0 \in x_m(0)$ for all $m \leq n$. By Lemma 2.4, $x'_m(0) > 1$ if $P(0,0) = C'(0)$ and $P_1(0,0)+P_2(0,0) > -mP_1(0,0)+C''(0)$, which implies that a non-trivial symmetric equilibrium exists for $m \leq n$ firms. Now, it remains to show that the rest of $n - m$ firms best react by producing nothing. Suppose that in equilibrium, the m firms produce a total output of $z_m > 0$. By hypothesis, $0 \in x_n(0)$, which implies that $xP(x, 0) \leq C(x)$ for all $x \in [0, K]$. By (A1), $xP(x + z_m, 0) < C(x)$ for all $x \in [0, K]$, which implies that $0 \in x(z_m, 0)$. Moreover, 0 is a strictly dominant strategy and $x(z_m, 0) = 0$, which is what we wanted to show.

(iii) For now, let us assume that there are $m \leq n$ firms. The condition in this part implies that $\Pi_1(z, s) \geq 0$ for some $s \in (0, K]$ and for all $z \leq ms$, i.e., there exist $s \in (0, K]$ and $z' \geq ms$ such that $\Pi(z', s) \geq \Pi(z, s)$ for all $z \leq ms$. Hence, the largest argmax of $\Pi(z, s)$, say z^*, must be grater or equal than ms, i.e., $z^* \geq ms$ and $z^*/m \geq s$. By Lemma 2.22, $z^*/m \in x_m(s)$ so there is an $s \in (0, K]$ such that an element of $x_m(s)$ is greater or equal than s. By the argument in part (ii), it follows that a non-trivial symmetric equilibrium exists for m firms. The proof that the rest of the $n-m$ firms best react by staying out of the market corresponds to that of part (ii). $\square$

Proof of Theorem 2.6.

(i) By the proof of Theorem 2.1, we know that $\bar{z}_n(s) = n\bar{x}_n(s)$ is increasing in s and by Amir and Lambson (2000), increasing in n for every s. Thus, by Tarski's fixed point Theorem (1955) and Milgrom and Roberts (1990), its maximal fixed point, $\bar{z}_n$, exists and is increasing in n. A similar argument is used to prove that $\underline{z}_n$ exists and is increasing in n.

(ii) First notice that $\Delta_3(z) = dP(z, z/n)/dz$, then, $\Delta_3(z) \leq 0$ implies that $P(z, z/n)$ is decreasing in z. Thus we have, $\bar{P}_n = P(\underline{z}_n, \underline{z}_n/n)$. Taking derivative with respect to n and reordering terms, we have:

$$\frac{d\bar{P}_n}{dn} = \Delta_3(\underline{z}_n)\frac{d\underline{z}_n}{dn} - \frac{P_2(\underline{z}_n, \underline{z}_n/n)\underline{z}_n}{n^2}.$$

By assumption and part (i), the first term of the previous expression is negative on $[\underline{z}_n, \underline{z}_{n+1}]$. Similarly, the second term is negative by (A2), hence, the minimal equilibrium price decreases with n. A similar argument shows that $\underline{P}_n$ is decreasing in n when $\Delta_3(\cdot) \leq 0$ on $[\bar{z}_n, \bar{z}_{n+1}]$. $\square$

Proof of Proposition 2.7. If an m-active-firm symmetric equilibrium exists (with positive production), we know that a (non-trivial) $m+1$ symmetric equilibrium exists with $\bar{z}_m \leq \bar{z}_{m+1}$, by Theorems 2.1 and 2.6. A m-active-firm symmetric equilibrium imply that $0 = \pi(0, \bar{z}_m, 0) \geq \pi(x, \bar{z}_m, 0)$ for all $x \in [0, K]$. Hence, by (A1) and $\bar{z}_m \leq \bar{z}_{m+1}$, $0 \geq xP(x + \bar{z}_m, 0) - C(x) \geq xP(x + \bar{z}_{m+1}, 0) - C(x)$ for all $x \in [0, K]$, i.e., $\pi(0, \bar{z}_{m+1}, 0) \geq \pi(x, \bar{z}_{m+1}, 0)$ for all $x \in [0, K]$, which implies that the rest of the $n-(m+1)$ firms best respond with zero production. Thus, a $(m+1)$-active-firm symmetric equilibrium exists with aggregated equilibrium output $\bar{z}_{m+1}$, which is greater than that for the m-active-firm symmetric equilibrium, $\bar{z}_m$. □

Proof of Theorem 2.8. At any interior equilibrium, $\bar{x}_n$ must satisfy the first order condition

$$P(n\bar{x}_n, \bar{x}_n) + \bar{x}_n P_1(n\bar{x}_n, \bar{x}_n) - C'(\bar{x}_n) = 0,$$

which implies that it is the maximal fixed point of the function

$$F(x; n) = -\frac{P(nx, x) - C'(x)}{P_1(nx, x)}.$$

Notice that

$$\frac{\partial F(x; n)}{\partial n} = -\frac{x}{P_1^2}\Delta_4(x).$$

Thus, by Milgrom and Roberts (1990), the direction of change in $\bar{x}_n$ when n increases is given by the opposite sign of $\Delta_4(x)$. □

Proof of Theorem 2.9.

(i) Consider the following inequalities

$$\pi_{n+1} = \bar{x}_{n+1} P(\bar{x}_{n+1} + \bar{y}_{n+1}, \bar{x}_{n+1}) - C(\bar{x}_{n+1})$$

$$\geq \bar{x}_n P(\bar{x}_n + \bar{y}_{n+1}, \bar{x}_{n+1}) - C(\bar{x}_n)$$

$\geq \bar{x}_n P(\bar{x}_{n+1} + \bar{y}_{n+1}, \bar{x}_{n+1}) - C(\bar{x}_n)$

$\geq \bar{x}_n P(\bar{x}_n + \bar{y}_n, \bar{x}_n) - C(\bar{x}_n) = \pi_n.$

The first inequality follows by equilibrium. The second one, by (A1), $P_1(z,s) < 0$, and $\bar{x}_{n+1} \geq \bar{x}_n$. The last one, by the assumption that $P(\bar{x}_{n+1} + \bar{y}_{n+1}, \bar{x}_{n+1}) \geq P(\bar{x}_n + \bar{y}_n, \bar{x}_n)$.

(ii) Consider the following

$\pi_n = \bar{x}_n P(\bar{x}_n + \bar{y}_n, \bar{x}_n) - C(\bar{x}_n)$

$\geq \bar{x}_{n+1} P(\bar{x}_{n+1} + \bar{y}_n, \bar{x}_n) - C(\bar{x}_{n+1})$

$\geq \bar{x}_{n+1} P(\bar{x}_{n+1} + \bar{y}_n, \bar{x}_{n+1}) - C(\bar{x}_{n+1})$

$\geq \bar{x}_{n+1} P(\bar{x}_n + \bar{y}_n, \bar{x}_{n+1}) - C(\bar{x}_{n+1})$

$\geq \bar{x}_{n+1} P(\bar{x}_{n+1} + \bar{y}_{n+1}, \bar{x}_{n+1}) - C(\bar{x}_{n+1}) = \pi_{n+1}.$

The first inequality follows by equilibrium. The second one, by (A1), $P_2(z,s) > 0$, and the assumption that $\bar{x}_{n+1} \leq \bar{x}_n$. The third one, by $P_1(z,s) < 0$ and $\bar{x}_{n+1} \leq \bar{x}_n$; the last one, by (A1) and Theorem 2.6 (i). □

Proof of Proposition 2.10.

(i) This part follows immediately from Theorem 2.6 part (i).

(ii) Consider the following inequalities

$CS_{n+1} - CS_n = \int_0^{\bar{z}_{n+1}} \{P(t, \bar{x}_{n+1}) - P(\bar{z}_{n+1}, \bar{x}_{n+1})\}dt - \int_0^{\bar{z}_n} \{P(t, \bar{x}_n) - P(\bar{z}_n, \bar{x}_n)\}dt$

$\geq \int_0^{\bar{z}_n} \{P(t, \bar{x}_{n+1}) - P(\bar{z}_{n+1}, \bar{x}_{n+1}) - P(t, \bar{x}_n) + P(\bar{z}_n, \bar{x}_n)\}dt$

$\geq \int_0^{\bar{z}_n} \{P(t, \bar{x}_{n+1}) - P(\bar{z}_n, \bar{x}_{n+1}) - P(t, \bar{x}_n) + P(\bar{z}_n, \bar{x}_n)\}dt$

$\geq 0.$

The first inequality follows by $P_1(z,s) < 0$ and $\bar{z}_{n+1} \geq \bar{z}_n$ ((A1) and Theorem 2.6 (i), respectively). The same results lead to the second inequality. The last inequality holds by hypotheses of the theorem, $\bar{x}_{n+1} \geq \bar{x}_n$ and $P_{12}(z,s) \leq 0$ for all z, s gives us $P(\bar{z}_n, \bar{x}_n) - P(t, \bar{x}_n) \geq P(\bar{z}_n, \bar{x}_{n+1}) - P(t, \bar{x}_{n+1})$ for all $t \leq \bar{z}_n$, which leads to the result.

(iii) The proof of this part is similar to that of part (ii). □

Proof of Proposition 2.11.

(i) This parts follows directly from Theorem 2.9 part (i).

(ii) Consider the following

$$(n+1)\pi_{n+1} - n\pi_n = \bar{z}_{n+1}[P(\bar{z}_{n+1}, \bar{x}_{n+1}) - A(\bar{x}_{n+1})] - \bar{z}_n[P(\bar{z}_n, \bar{x}_n) - A(\bar{x}_n)]$$

$$\geq \bar{z}_n[P(\bar{z}_{n+1}, \bar{x}_{n+1}) - A(\bar{x}_{n+1})] - \bar{z}_n[P(\bar{z}_n, \bar{x}_n) - A(\bar{x}_n)]$$

$$= \bar{z}_n\left\{[P(\bar{z}_{n+1}, \bar{x}_{n+1}) - P(\bar{z}_n, \bar{x}_n)] - [A(\bar{x}_{n+1}) - A(\bar{x}_n)]\right\} \geq 0.$$

The first inequality follows by Theorem 2.6 part (i) and $P(\bar{z}_{n+1}, \bar{x}_{n+1}) - A(\bar{x}_{n+1}) \geq 0$; the second one, by assumption.

(iii) Consider the following steps

$$\pi_n = \bar{x}_n\left\{P(n\bar{x}_n, \bar{x}_n) - A(\bar{x}_n)\right\}$$

$$\geq \tfrac{n+1}{n}\bar{x}_{n+1}\left\{P\left[\tfrac{n+1}{n}\bar{x}_{n+1} + (n-1)\bar{x}_n, \bar{x}_n\right] - A(\tfrac{n+1}{n}\bar{x}_{n+1})\right\}$$

$$\geq \tfrac{n+1}{n}\bar{x}_{n+1}\left\{P\left[\tfrac{n+1}{n}\bar{x}_{n+1} + (n-1)\tfrac{n+1}{n}\bar{x}_{n+1}, \bar{x}_n\right] - A(\tfrac{n+1}{n}\bar{x}_{n+1})\right\}$$

$$= \tfrac{n+1}{n}\bar{x}_{n+1}\left\{P\left[(n+1)\bar{x}_{n+1}, \bar{x}_n\right] - A(\tfrac{n+1}{n}\bar{x}_{n+1})\right\}$$

$$\geq \tfrac{n+1}{n}\bar{x}_{n+1}\left\{P\left[(n+1)\bar{x}_{n+1}, \bar{x}_{n+1}\right] - A(\bar{x}_{n+1})\right\}$$

$$= \tfrac{n+1}{n}\pi_{n+1}.$$

The first inequality follows by optimality. The second one, by $(n+1)\bar{x}_{n+1} \geq n\bar{x}_n$ (Theorem 2.6 part (i)) and (A1), $P_1(z,s) < 0$. The last inequality holds by the assumptions that $\bar{x}_{n+1} \leq \bar{x}_n$, $A(\frac{n+1}{n}\bar{x}_{n+1}) \leq A(\bar{x}_{n+1})$, and $P_2(z,s) > 0$. □

Proof of Proposition 2.12.

(i) Consider the following relations

$W_{n+1} - W_n = \int_0^{\bar{z}_{n+1}} P(t,\bar{x}_{n+1})dt - \bar{z}_{n+1}A(\bar{x}_{n+1}) - \int_0^{\bar{z}_n} P(t,\bar{x}_n)dt + \bar{z}_n A(\bar{x}_n)$

$\geq \int_0^{\bar{z}_n}\{P(t,\bar{x}_{n+1}) - A(\bar{x}_{n+1})\}dt - \int_0^{\bar{z}_n}\{P(t,\bar{x}_n) - A(\bar{x}_n)\}dt$

$\geq 0.$

The first inequality is given by the fact that $P(t,\bar{x}_{n+1}) - A(\bar{x}_{n+1}) \geq 0$ for all $t \leq \bar{z}_{n+1}$, and $\bar{z}_{n+1} \geq \bar{z}_n$ (Theorem 2.6). The second one, by hypothesis of the result.

(ii) First notice that the social welfare function at any production level x and expected size of the network s, when there are n symmetric firms is given by

$$V_n(x,s) = \int_0^{nx} P(t,s)\ dt - nC(x).$$

$V_n(x,s)$ is a concave function with respect to x since $\frac{\partial^2 V_n(x,s)}{\partial x^2} = n[nP_1(nx,s) - C''(x)] < 0$, by (A1) and (A4). Now, consider the following relations

$W_{n+1} - W_n = \int_0^{(n+1)\bar{x}_{n+1}} P(t,\bar{x}_{n+1})dt - (n+1)C(\bar{x}_{n+1}) - \left[\int_0^{n\bar{x}_n} P(t,\bar{x}_n)dt - nC(\bar{x}_n)\right]$

$\geq \pi_{n+1} + \int_0^{n\bar{x}_{n+1}} P(t,\bar{x}_{n+1})dt - nC(\bar{x}_{n+1}) - \left[\int_0^{n\bar{x}_n} P(t,\bar{x}_{n+1})dt - nC(\bar{x}_n)\right]$

$\geq V_n(\bar{x}_{n+1},\bar{x}_{n+1}) - V_n(\bar{x}_n,\bar{x}_{n+1})$

$\geq \frac{\partial V_n(\bar{x}_{n+1},\bar{x}_{n+1})}{\partial x}(\bar{x}_{n+1} - \bar{x}_n)$

$= n\left[P(n\bar{x}_{n+1},\bar{x}_{n+1}) - C'(\bar{x}_{n+1})\right](\bar{x}_{n+1} - \bar{x}_n)$

$\geq n\left[P((n+1)\bar{x}_{n+1},\bar{x}_{n+1}) - C'(\bar{x}_{n+1})\right](\bar{x}_{n+1} - \bar{x}_n)$

≥ 0.

The first inequality follows by $\int_0^{(n+1)\bar{x}_{n+1}} P(t, \bar{x}_{n+1})dt \geq \int_0^{n\bar{x}_{n+1}} P(t, \bar{x}_{n+1})dt + \bar{x}_{n+1}P(\bar{z}_{n+1}, \bar{x}_{n+1})$, the assumption that $\bar{x}_{n+1} \geq \bar{x}_n$ and (A1). The second one, by $\pi_{n+1} \geq 0$. The third inequality holds by concavity of $V_n(\cdot, s)$. The next one, by (A1); and the last one, by the Cournot property and the assumption that $\bar{x}_{n+1} \geq \bar{x}_n$. □

Proof of Lemma 2.13. Producing zero is a FECE for the oligopoly with compatible networks, $0 \in x_n^C(0)$, if and only if each firm is better off by producing nothing than any positive output (less than K), i.e., if and only if $0 \geq xP(x, 0) - C(x)$ $\forall x \in [0, K]$, which is equivalent to $0 \in x_n^I(0)$. □

Proof of Lemma 2.14. If K is a symmetric equilibrium for the n-oligopoly with complete incompatibility we have

$$KP(nK, K) - C(K) \geq xP(x + (n-1)K, K) - C(x) \text{ for all } x \in [0, K).$$

The facts that $x < K$, $K < nK$, for $n > 1$, and that the profit function $\pi(x, s)$ satisfies the single crossing property in (x, s), by Lemma 2.19, give us

$$KP(nK, nK) - C(K) \geq xP(x + (n-1)K, nK) - C(x) \text{ for all } x \in [0, K),$$

which implies that K is an equilibrium for the n-oligopoly with complete compatibility. □

Proof of Theorem 2.15.

(i) If $\bar{x}_n^I = 0$, the result trivially holds.

Now suppose that $\bar{x}_n^I$ is an interior equilibrium. The only possibility that we have to rule out, is that the trivial equilibrium is the unique one for complete compatibility. Suppose it is, then, 0 is a strictly dominant strategy for the game with complete compatibility, i.e., $xP(x+y,0) < C(x)$ for all $x, y > 0$, but this condition implies that 0 is also the unique equilibrium for the game with complete incompatibility, which contradicts the assumption that $\bar{x}_n^I$ is interior. Then, $\bar{x}_n^C$ is interior or K. Let us look at the interesting case when it is interior, otherwise, the result trivially holds.

Any interior solution x_n^C must satisfy the first order condition

$$P(nx_n^C, nx_n^C) + x_n^C P_1(nx_n^C, nx_n^C) - C'(x_n^C) = 0.$$

Similarly, any interior solution x_n^I satisfies the first order condition

$$P(nx_n^I, x_n^I) + x_n^I P_1(nx_n^I, x_n^I) - C'(x_n^I) = 0.$$

Let us define the following function $F_t : [0, K] \to \Re$ by

$$F(x; t) = -\frac{P(nx, ntx) - C'(x)}{P_1(nx, ntx)},$$

where $t \in \Re$ denotes a parameter. For this part, we focus on the values of $x \in [0, K]$ such that $F(x; t) \geq 0$, thus, we have $P(nx, ntx) - C'(x) \geq 0$ by (A1).

Notice that when $t = 1$, a fixed point of the function F gives us a FECE for the oligopoly with n firms and complete compatibility. Similarly, when $t = 1/n$, a fixed point of F can be interpreted as a FECE for an oligopoly with n firms and complete incompatibility. In fact, we know that in both cases, $t = 1$ and $t = 1/n$, we have fixed points because we are in the case where solutions are interior.

Moreover,

$$\frac{\partial F_t(x)}{\partial t} = -\frac{nx}{P_1^2(nx,ntx)}\{P_1(nx,ntx)P_2(nx,ntx) - [P(nx,ntx) - C'(x)]P_{12}(nx,ntx)\}$$
$$= -\frac{nx}{P_1^2(nx,ntx)}\{-[P(nx,ntx)P_{12}(nx,ntx) - P_1(nx,ntx)P_2(nx,ntx)]$$
$$+C'(x)P_{12}(nx,ntx)\} \geq 0.$$

The previous result can be easily seen by cases. First, if $P_{12}(nx,ntx) \geq 0$, the right hand side of the first equality gives us the result. By (A1),

$$P_1(nx,ntx)P_2(nx,ntx) < 0,$$

and because $P(nx,ntx) - C'(x) \geq 0$, we have that

$$[P(nx,ntx) - C'(x)]P_{12}(nx,ntx) \geq 0,$$

thus, the derivative is grater or equal than zero.

Second, if $P_{12}(nx,ntx) \leq 0$, the result follows from the right hand side of the second equality. By (A5), log supermodularity of $P(z,s)$, we have that

$$P(nx,ntx)P_{12}(nx,ntx) - P_1(nx,ntx)P_2(nx,ntx) > 0,$$

and by (A2), $C'(x) \geq 0$ which leads to the result.

The previous inequality implies that the extremal fixed points of $F(\cdot)$ increase in t, by Milgrom and Roberts (1990), thus $\bar{x}_n^C \geq \bar{x}_n^I$.

Finally, if K is the highest equilibrium for the n-oligopoly with complete incompatibility, K is also the highest equilibrium for the n-oligopoly with complete compatibility, by Lemma 2.14.

By the previous arguments, the proof for this part is complete.

(ii) Consider the following inequalities

$$
\begin{aligned}
P_n^C &= P(n\bar{x}_n^C, n\bar{x}_n^C) \\
&\geq P(n\bar{x}_n^I, n\bar{x}_n^I) \\
&\geq P(n\bar{x}_n^I, \bar{x}_n^I).
\end{aligned}
$$

The first inequality follows by the assumption that $\Delta_5(\cdot) \geq 0$ and Theorem 2.15 part (i); the second one, by the assumption that $P_2(z, s) > 0$. □

Proof of Theorem 2.16.

(i) Consider the following inequalities

$$
\begin{aligned}
\pi_n^C &= \bar{x}_n^C P(\bar{x}_n^C + \bar{y}_n^C, \bar{x}_n^C + \bar{y}_n^C) - C(\bar{x}_n^C) \\
&\geq \bar{x}_n^I P(\bar{x}_n^I + \bar{y}_n^C, \bar{x}_n^C + \bar{y}_n^C) - C(\bar{x}_n^I) \\
&\geq \bar{x}_n^I P(\bar{x}_n^C + \bar{y}_n^C, \bar{x}_n^C + \bar{y}_n^C) - C(\bar{x}_n^I) \\
&\geq \bar{x}_n^I P(\bar{x}_n^I + \bar{y}_n^I, \bar{x}_n^I + \bar{y}_n^I) - C(\bar{x}_n^I) \\
&\geq \bar{x}_n^I P(\bar{x}_n^I + \bar{y}_n^I, \bar{x}_n^I) - C(\bar{x}_n^I) \\
&= \pi_n^I.
\end{aligned}
$$

The first inequality follows by Cournot property; the second one, by (A1), $P_1(z, s) < 0$, and part (i) Theorem 2.15, $\bar{x}_n^C \geq \bar{x}_n^I$; the third one is true by the assumption that $\Delta_5(\cdot) \geq 0$ and $\bar{x}_n^C \geq \bar{x}_n^I$, and the last one, by (A1), $P_2(z, s) > 0$.

(ii) Consider the following inequalities

$$
\begin{aligned}
CS_n^C - CS_n^I &= \int_0^{n\bar{x}_n^C} [P(t, n\bar{x}_n^C) - P(n\bar{x}_n^C, n\bar{x}_n^C)]\, dt - \int_0^{n\bar{x}_n^I} [P(t, \bar{x}_n^I) - P(n\bar{x}_n^I, \bar{x}_n^I)]\, dt \\
&\geq \int_0^{n\bar{x}_n^I} [P(t, n\bar{x}_n^C) - P(n\bar{x}_n^C, n\bar{x}_n^C)]\, dt - \int_0^{n\bar{x}_n^I} [P(t, \bar{x}_n^I) - P(n\bar{x}_n^I, \bar{x}_n^I)]\, dt
\end{aligned}
$$

$$\geq \int_0^{n\bar{x}_n^I}[P(t, n\bar{x}_n^C) - P(n\bar{x}_n^C, n\bar{x}_n^C)]\ dt - \int_0^{n\bar{x}_n^I}[P(t, \bar{x}_n^I) - P(n\bar{x}_n^C, \bar{x}_n^I)]\ dt \geq 0.$$

The first inequality follows by Theorem 2.15 part (i) and $P_1(z, s) < 0$. Under the assumption that $P_n^I \geq P_n^C$ at the highest equilibrium, the second line becomes positive given that $n\bar{x}_n^C \geq \bar{x}_n^I$ and $P_2(z, s) < 0$, which proves the first part of the result. For the second part, notice that the second inequality follows from (A1), $P_1(z, s) < 0$, and the last one by the assumption that $P_{12}(z, s) \leq 0$. To see this, notice that $t \in [0, n\bar{x}_n^I]$ and $n\bar{x}_n^C \geq n\bar{x}_n^I$ imply that $t \leq n\bar{x}_n^C$, thus, adding the assumption that $P_{12}(z, s) \leq 0$ and the result that $n\bar{x}_n^C \geq \bar{x}_n^I$ imply that $P(t, n\bar{x}_n^C) - P(t, \bar{x}_n^I) \geq P(n\bar{x}_n^C, n\bar{x}_n^C) - P(n\bar{x}_n^C, \bar{x}_n^I)$ for all $t \in [0, n\bar{x}_n^I]$, which gives us the result.

(iii) Recall, by the proof of Proposition 2.12, that the social welfare function at production level x and expected size of the network s, with n symmetric firms is given by $V_n(x, s) = \int_0^{nx} P(t, s)\ dt - nC(x)$, which is a concave function with respect to x. Then, we have that at the highest equilibria

$$\begin{aligned}
W_n^C - W_n^I &= \left\{\int_0^{n\bar{x}_n^C} P(t, n\bar{x}_n^C)\ dt - nC(\bar{x}_n^C)\right\} - \left\{\int_0^{n\bar{x}_n^I} P(t, \bar{x}_n^I)\ dt - nC(\bar{x}_n^I)\right\} \\
&\geq \left\{\int_0^{n\bar{x}_n^C} P(t, n\bar{x}_n^C)\ dt - nC(\bar{x}_n^C)\right\} - \left\{\int_0^{n\bar{x}_n^I} P(t, n\bar{x}_n^C)\ dt - nC(\bar{x}_n^I)\right\} \\
&= V_n(\bar{x}_n^C, n\bar{x}_n^C) - V_n(\bar{x}_n^I, n\bar{x}_n^C) \\
&\geq \tfrac{\partial V_n(\bar{x}_n^C, n\bar{x}_n^C)}{\partial x}(\bar{x}_n^C - \bar{x}_n^I) \\
&= n[P(n\bar{x}_n^C, n\bar{x}_n^C) - C'(\bar{x}_n^C)](\bar{x}_n^C - \bar{x}_n^I) \geq 0
\end{aligned}$$

The fist inequality follows by (A1), $P_2(z, s) > 0$; the second one, by concavity of $V_n(\cdot, s)$, and the last one, because $P(n\bar{x}_n^C, n\bar{x}_n^C) \geq C'(\bar{x}_n^C)$ and $\bar{x}_n^C \geq \bar{x}_n^I$, by Theorem 2.15 part (i). □

Proof of Corollary 2.17.

Amir and Lazzati (2011) show that under our assumptions, no asymmetric equilibria exist in the compatible industry. Thus, the assumption that $\bar{z}_n^C = \bar{z}_n^I$ implies that $\bar{z}_n^C = n\bar{x}_n^C = \sum_{i=1}^n \bar{x}_{in}^I$ and $\bar{z}_n^C \geq \bar{x}_{in}^I$ for all $i = 1, ..., n$. Let us order the incompatible firms by price, i.e., $P_1^I \geq P_2^I \geq ... \geq P_n^I$, and define $\bar{x}_{0n} \equiv 0$, then

$$CS_n^I - CS_n^C = \sum_{i=1}^n \left\{ \int_{\bar{x}_{(i-1)n}^I}^{\bar{x}_{in}^I} [P(t, \bar{x}_{in}^I) - P(\bar{z}_n^I, \bar{x}_{in}^I)] \, dt \right\} - \int_0^{n\bar{x}_n^C} [P(t, \bar{z}_n^C) - P(\bar{z}_n^C, \bar{z}_n^C)] \, dt$$

$$CS_n^I - CS_n^C = \sum_{i=1}^n \left\{ \int_{\bar{x}_{(i-1)n}^I}^{\bar{x}_{in}^I} [P(t, \bar{x}_{in}^I) - P(t, \bar{z}_n^C) - P(\bar{z}_n^I, \bar{x}_{in}^I) + P(\bar{z}_n^C, \bar{z}_n^C)] \, dt \right\},$$

which is positive by the assumption that $P_{12}(z, s) \geq 0$. □

CHAPTER 3
ENVIRONMENTAL REGULATION OF OLIGOPOLIES: EMISSION VERSUS PERFORMANCE STANDARDS

3.1 Introduction

The rapid growth of industrial production in the XX century has raised numerous questions about its consequences for the natural environment. Only in the second half of the same century, however, the issue of pollution has become sufficiently alarming to make the policy makers seek the means towards the efficient conservation of the environment. With this purpose numerous environmental policy instruments have been studied, providing foundations for the establishment of environmental rankings comparing the performance of such instruments. In this study, we examine a partial equilibrium model that analyzes the performance of two command-and-control policy instruments: the emission and performance standards. Unlike the vast majority of the models, we assess the incentives to invest in R&D under imperfectly competitive conditions in the output market. In that way, our approach highlights the importance that the strategic interactions of oligopolistic firms have for the R&D incentives. Additionally, by considering the objective of the regulator, who seeks to maximize social welfare, this analysis provides a welfare comparisons that, to our knowledge, have not been offered previously.

The literature analyzing the performance of environmental policy instruments tends to neglect the command-and-control apparatus. The reason behind it is that

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BASED ON JOINT WORK WITH KATARZYNA WERNER.

the command-and-control instruments involve a presence of a regulatory authority that imposes restrictions on the emission levels produced by firms, ipso facto, restricting these firms' freedom to decide about their abatement levels. As a result only a few papers account for either the emission standard or the performance standard, and even fewer make comparisons between both instruments. Among the former ones, e.g., Milliman and Prince (1989) find that the emission standard is weaker at promoting technological change than market based instruments such as subsidies, taxes and permits. Requate (2005) obtains a different result and demonstrates that an emission standard is not necessarily characterized by the lowest innovation incentives, despite the opposing view presented in the literature. Strongly contrasting the latter, Wenders (1975) shows that an emission standard may provide no economic inducement to produce any innovations in pollution abatement. Neither of these authors, however, considers the performance standard.

Despite being frequently applied by the regulators (see Bruneau, 2004, p.1194 and Requate, 2005, p. 178) so far the performance standard has been rarely accounted for in the theoretical literature on emission reduction. One reason for this shortcoming is the negligence of the output market in this literature. An example of such negligence is given by Downing and White (1986), Malueg (1989), Milliman and Prince (1989) and Jung et al. (1996) who assess environmental policy instruments based on the incentives they provide to adopt less polluting technology. These authors measure the incentives using the aggregate cost savings of the industry while ignoring the output effect derived from the lower abatement costs (Montero, 2002;

Bruneau, 2004; Requate, 2005). Discarding firms' decisions on the quantity of output being produced rules out the possibility of modeling the performance standard. Parry (1998) recognizes this limitation and employs the analysis of R&D incentives in the competitive output market. The results of his analysis comply with the earlier findings highlighting the superiority of the market based instruments. Montero (2002) obtains similar results investigating the model of perfect competition in which firms can reduce their compliance costs by investing in the environmental R&D. In the same framework he examines the R&D incentives under emission standard and finds that this command-and-control instrument performs significantly better than the performance standard. By deriving opposite implications Bruneau (2004) strongly contradicts Montero's findings. He explains that under the performance standard a firm that has a higher initial output but attains the same level of emissions like under the emission standard will have higher abatement costs. From this fact he immediately infers that the reduction in the abatement costs is larger under the performance standard. Bruneau (2004), however, makes no comparison of both instruments under imperfect output competition.

Montero (2002) fills this gap and investigates the Cournot duopoly model of imperfect competition in the output market. Unlike under perfect competition, the investigation focuses not only on the cost-minimizing (direct) effect of R&D investment, but on the strategic effect, too. The strategic effect is said to reflect the impact of one firm's choice of R&D level on the output decision of another firm. In the two-stage model featuring both effects the level of emissions is taken as given and the marginal production cost is implicitly assumed to be constant. Solving for the profit maximizing levels of R&D investment and output, Montero does not obtain a clear-cut result of the intra-comparison between the standards. The reason for it is that the cost savings from innovation (the direct effect) are larger under emission standard, but the profits (the strategic effect) are higher under performance standard. Hence, Montero conditions the final outcome of the comparison on the level of demand for output and the cost of R&D. In particular, he shows that the higher the cost of R&D and the less elastic output demand are, the more likely the performance standard is to offer superior R&D incentives. Montero, however, does not verify whether this result is robust also when the regulator chooses a socially optimal level of emission in the first instance.

This is where we offer our contribution. Instead of adopting a given emission target, our model incorporates a damage function, whose presence enables to determine a socially optimal level of emission. As a result, the game under consideration involves three instead of two stages, with the regulatory body selecting the socially optimal level of emissions in the first stage. The assumptions of imperfect competition in the output market and constant marginal production costs are maintained. Additionally, following the paper of d'Aspremont and Jacquemin (1988), we adopt a quadratic form of the R&D cost to account for diminishing returns to R&D expenditures. To simplify the exposition further, particular functional forms have been assigned to the remaining variables of interest, including demand and abatement cost.

In such a framework we find a symmetric subgame perfect equilibrium (SPE) under each policy instrument. This equilibrium is decomposed of the socially optimal level of emission (or ratio of emissions per output under the performance standard), the R&D investment, and the output level. Based on the comparison between both equilibria we establish that social welfare is larger under performance standard for the given set of primitives. This result may explain why the performance standard is commonly applied in real life. From the theoretical point of view, however, this result cannot be easily clarified. The reason is that in our model the conventional link between the R&D investment and welfare level is missing. In fact, according to our model, the level of welfare is independent of the R&D investment under both regimes. Hence, the explanation relying on the simple argument positively relating the level of welfare to the level of R&D investment fails.

We also find that the nature of a policy instrument is crucial in determining the R&D incentives that this instrument provides. In particular, when the levels of emissions are endogenous under both policy instruments (hence, when the welfare is maximized), larger R&D incentives are obtained under emission standard. However, when these levels are fixed to be equal, it is the performance standard that yields larger incentives to invest in R&D. As a result, the comparison of R&D incentives remains inconclusive. In that sense, our results resemble those of Montero (2002) - despite using a different methodology than this author we find that the final result of the comparison of R&D incentives is governed by the underlying assumptions.

The rest of the paper is organized as follows. The next section sets out the basic model and clarifies the assumptions behind it. Sections 3.2.1 and 3.2.2 provide analysis of equilibrium under the emission and the performance standards, respectively. Section 3.2.3 presents some comparative results, while Section 3.3 concludes. All proofs are given in Section 3.4.

3.2 The model

Consider a duopoly market with three agents: two ex-ante symmetric firms pursuing the profit maximizing objective and a regulator. The firms produce a homogenous good for which the (inverse) demand is given by function P, such that $P : [0, \infty) \to [0, \infty)$. A quantity produced by a single firm is denoted by q, while the aggregate quantity produced in this market is given by Q. The production process itself is costless, but entails harmful emissions damaging the environment. The damage takes the form of function D, where $D : [0, \infty) \to [0, \infty)$. Function D is increasing in the number of total emissions, E, such that $D'(\cdot) > 0$.

Unlike firms, the environmental regulator is concerned with the maximization of social welfare. Hence, it employs various policy instruments in order to control the level of emissions. In this framework two such instruments are considered: the emission and performance standards. Emission standard constraints the level of emissions generated by a single firm, e, while performance standard imposes a restriction on the ratio of emissions per output, h. Once the chosen standard is exceeded, the emitting firm becomes a subject to abatement cost C, where $C : [0, \infty) \times [0, \infty) \to [0, \infty)$. The abatement cost $C(y, x)$ increases in the number of units of emissions abated, $y = q - e$ for emission standard and $y = q(1 - h)$ for performance standard, and decreases in the R&D investment, x, such that $C_1(\cdot,\cdot) > 0$ and $C_2(\cdot,\cdot) < 0$. The cost can be reduced if the firm chooses to improve its abatement technology. This, however, requires investing in R&D, which is also costly. In this framework the cost of R&D investment is given by function g and is quadratic to reflect the diminishing returns to R&D expenditures (see D'Aspremont and Jacquemin, 1988). Additionally, market structure (P, C, D, g) is characterized as follows:

(A1) the inverse demand function is linear, $P(Q) = a - bQ$, where $a, b > 0$;

(A2) the abatement cost function is given by $C(y, x) = (c - x)y$, where $c > 0$;

(A3) the R&D cost function is $g(x) = \gamma x^2/2$, where $\gamma > 0$;

(A4) the damage function is given by $D(E) = sE^2/2$, where $s > 0$;

(A5) $9b\gamma > 8$, $9b + 8s < 18b\gamma s$, $4a < 9bc\gamma$, $a < 2\gamma s(a - c)$, $9bc < 8s(a - c)$ and $a > 9bc\gamma - 6\gamma s(a - c)$.

In such a three-stage game the regulator concerned with the level of social welfare moves first by imposing caps on the emission levels/ratio of emissions per output for each firm. The firms move next by selecting simultaneously the level of R&D investment in response to the imposed emission ceilings. Finally, these firms compete in output (à la Cournot).

Using backwards induction we solve the subgame perfect equilibrium (SPE) of the game, entailing optimal levels of e (Section 3.2.1) and h (Section 3.2.2).

3.2.1 Emission Standards

In this model, the regulator establishes a cap on the number of emissions that a firm can generate by its production. Since the firms are symmetric, we assume that the regulator sets the same emissions ceiling, e, for both firms. In the last stage of the game, firm i chooses its output, q_i, and the final level of emissions (after abatement), e_i, such that it maximizes its profit given the level of e, $x_i < c$, and its rival's production q_j. In this way, the total number of emissions abated by the firm equals $q_i - e_i$. Hence, the optimization problem of firm i in this stage is given by

$$\max_{q_i, e_i} q_i[a - b(q_i + q_j)] - (c - x_i)(q_i - e_i) \text{ s.t. } e_i \leq e. \tag{3.1}$$

Since the objective function in (3.1) is increasing in e_i, the firm chooses $e_i = e$ and the firm's maximization problem reduces to

$$\max_{q_i} q_i[a - b(q_i + q_j)] - (c - x_i)(q_i - e), \tag{3.2}$$

which leads to equilibrium individual outputs $q_i(x_i, x_j)$, for $i, j \in \{1, 2\}$ and $i \neq j$[1].

In the second stage, the firms simultaneously choose their levels of R&D, x_1 and x_2, given e and the equilibrium in the last stage. Thus, firm i solves the following problem

$$\max_{x_i} q_i(x_i, x_j)\,[a - b(q_i(x_i, x_j) + q_j(x_i, x_j))] - (c - x_i)[q_i(x_i, x_j) - e] - \frac{\gamma x_i^2}{2}. \tag{3.3}$$

Notice that an additional term is present in (3.3). This term denotes the cost that

[1]In a general setting, the equilibrium in this stage also depends on e, i.e., $q_i(x_i, x_j, e)$. Our particular assumptions lead to an equilibrium that does not depend on e in this stage.

firm i incurs when investing the amount of x_i in R&D.

By symmetry of the game, this stage leads to symmetric equilibrium $x(e)$ and $q(e)$.

Finally, in the first stage, the regulator chooses the emission cap for each firm given the symmetric equilibrium of the second stage. Hence, the regulator solves

$$\max_e \int_0^{2q(e)} P(t)dt - 2[c - x(e)][q(e) - e] - \frac{s(2e)^2}{2} - \gamma[x(e)]^2, \qquad (3.4)$$

where the industry emissions are equal to $E = 2e$.

The first term in the welfare maximization problem corresponds to the consumer surplus, while the second term reflects the costs of abatement incurred by both firms. The remaining terms illustrate the damage cost for the society, $\frac{s(2e)^2}{2}$, and the total cost of R&D, $\gamma[x(e)]^2$.

Explicitly, in the first stage the regulator solves

$$\max_e \left\{ \frac{e^2(9b + 8s - 18b\gamma s) + e(18bc\gamma - 8a) + 4\gamma(a - c)^2}{9b\gamma - 4} \right\}.$$

Now we are ready for our first result. Since the equilibrium is symmetric, we drop the sub-indexes in the variables of interest. The calculations are detailed in Section 3.4.

Proposition 3.1. *Under assumptions (A1)-(A5), a symmetric SPE exists with outcome* $e^* = \frac{4a - 9bc\gamma}{9b + 8s - 18b\gamma s}$, $x^* = \frac{9bc - 8s(a-c)}{9b + 8s - 18b\gamma s}$ *and* $q^* = \frac{3a - 6\gamma s(a-c)}{9b + 8s - 18b\gamma s}$. *Moreover,* $0 < x^* < c$ *and* $0 < e^* < q^*$.

In the next section, a similar game will be solved in the framework with the performance standard instead of the emission standard. Due to the complexity of the model under performance standard no close form-solution of the SPE can be computed. Instead, a numerical analysis is provided to emphasize the relevance of the endogeneity of the instruments.

3.2.2 Performance Standards

Performance standard refers to the cap, say h, that the regulator imposes on the proportion of emissions that a firm can emit per unit of production. For example, if the regulator chooses $h = 0.70$, the firm can pollute at most 70% of its production. Hence, if the production equals $q = 100$, the emissions must be reduced to 70, so that $q(1-h) = 30$ units of emissions must be abated.

Therefore, in the last stage, firm i solves

$$\max_{q_i} q_i[a - b(q_i + q_j)] - (c - x_i)q_i(1-h), \tag{3.5}$$

given h chosen by the regulator, x_i from the second stage, and the production of its rival, q_j. The equilibrium in this stage is described by $q_i(x_i, x_j, h)$ for firm $i \in \{1, 2\}$, $i \neq j$.

In the second stage, firm i solves

$$\max_{x_i} q_i(x_i, x_j, h)\left[a - b(q_i(x_i, x_j, h) + q_j(x_i, x_j, h))\right] - (c - x_i)q_i(x_i, x_j, h)(1-h) - \frac{\gamma x_i^2}{2}. \tag{3.6}$$

By symmetry, we obtain the equilibrium variables $x(h)$ and $q(h)$.

Under assumption 6, (A6), $8(1-h)^2 < 9b\gamma$, $a > c(1-h)$ and $9bc\gamma > 4a(1-h)$, so that the second stage equilibrium can be characterized in the following way:

Proposition 3.2. *Assume (A1)-(A4) and (A6). For all $0 < h < 1$, the equilibrium* *consists of* $x(h) = \frac{4(1-h)[c(1-h)-a]}{4(1-h)^2-9b\gamma}$ *and* $q(h) = \frac{3\gamma[c(1-h)-a]}{4(1-h)^2-9b\gamma}$*, with* $0 < x(h) < c$ *and* $q(h) > 0$*.*

In the first stage, the regulator chooses $0 < h < 1$ that solves the following optimization problem

$$\max_h \int_0^{2q(h)} P(t)dt - 2[c - x(h)]q(h)(1-h) - \frac{s[2hq(h)]^2}{2} - \gamma[x(h)]^2. \tag{3.7}$$

Using the results in Proposition 3.2 and equation (3.7), the regulator problem becomes

$$\max_h \frac{2\gamma[a - c(1-h)]^2[9\gamma(2b - h^2 s) - 8(h-1)^2]}{[4(1-h)^2 - 9b\gamma]^2}.$$

The first-order conditions of the previous problem lead to two real roots[2], out of which one is irrelevant as it induces zero production. The remaining root is the solution to our problem, however, it is analytically non-tractable. For this reason we present our results for a particular set of primitives. Specifically, we compare the equilibria under emission and performance standards for particular values of parameters a and γ, which denote the level of demand and the cost of R&D, respectively. We enable each of these parameters to vary, while the remaining variables b, c and s are normalized to 1. First, we assume that a changes while γ is fixed (Section 3.2.3.1); second, we vary the values of γ given a (Section 3.2.3.2).

Finally, in Section 3.2.3.3 we analyze the scenario in which one of the instruments is exogenous (is not chosen to maximize the social welfare function). In particular, the allowed level of emissions under emission standard is set to be equal to

[2] There are four roots in total, but two of them are imaginary ones.

the level of emissions endogenously generated under performance standard. Equalizing the levels of emissions under both policy instruments helps to compare our results with those of Montero (2002). In particular, the idea is to highlight some crucial differences in the equilibrium outcomes of the model when the standards are exogenous (as in Montero (2002)) and when they are not (this paper). Section 3.2.3.4 provides a deeper discussion of the results obtained in Sections 3.2.3.1-3.2.3.3.

3.2.3 Comparison for fixed $b = c = s = 1$

This section compares the equilibria of the two games for specific values of primitives. In particular, throughout this section it is assumed that $b = c = s = 1$. All results are obtained through a qualitative analysis of the equilibrium variables. Under emission standard, the equilibrium variables are given by Proposition 3.1. Under performance standard, the solutions and qualitative characteristics are obtained computationally. In both models, the outcomes of the game are symmetric and carry the superscript e or p, for the emission and performance standard, respectively. Further, W denotes total welfare, e^* stands for the equilibrium emission cap under emission standard and h^* denotes the equilibrium level of emission to output ratio under performance standard.

3.2.3.1 Fixing parameter $\gamma = 1.5$

In order to compare the R&D incentives offered under both policy instruments it is necessary to assess the impact of the R&D investment on output and welfare. Proposition 3 provides such an assessment for the specified values of parameters.

Proposition 3.3. *Let $b = c = s = 1$ and $\gamma = 1.5$. Then*

a) for $2.25 < a < 2.433$, $x^p > x^e$ and $q^p > q^e$;

b) for $2.433 < a < 2.979$, $x^e > x^p$ and $q^p > q^e$;

c) for $2.979 < a < 3.375$, $x^e > x^p$ and $q^e > q^p$;

d) $W^p > W^e$ for all $2.25 < a < 3.375$;

e) e^* and h^* are decreasing in a;

f) x^e, x^p, q^e, q^p, W^p and W^e are increasing in a for all $2.25 < a < 3.375$[3].

Interestingly, investing more in R&D does not lead to a larger output or a larger social welfare. This conjecture applies equally to both policy instruments under consideration. In fact, by Proposition 3.3 part (d), the social welfare is higher under performance standard, irrespective of the level of R&D or output. In particular, one can show that even when both, the levels of output and the R&D investment, are smaller under performance standard, the level of social welfare still dominates that under emission standard. For instance, when $a \in (2.979, 3.375)$, performance standard leads to a lower R&D investment ($x^e > x^p$) and lower level of individual output ($q^e > q^p$). Yet, the level of welfare is higher ($W^p > W^e$).

Table 3.1 summarizes these results. The abbreviation *e.s.* stands for emission standard and *p.s.* for performance standard.

The key assumption in Montero's (2002) analysis is that the emission levels under both policy instruments are equal, which significantly simplifies the comparison

[3]The condition that $a > 2.25$ is required for the optimal output under emission standard to be greater than the emissions level, $q^e > e^*$. Also, at $a = 3.375$, $e^* = h^* = 0$, $x^e = x^p = 1 = c$, $q^e = q^p = 1.125$ and $W^e = W^p = 3.5625$.

Variable	e.s.	p.s.	e.s.	p.s.	e.s.	p.s.
	$(a = 2.3)$		$(a = 2.5)$		$(a = 3.1)$	
Standard	0.43	0.65	0.35	0.59	0.11	0.31
R&D per firm	0.14	0.21	0.30	0.27	0.78	0.57
Individual output	0.48	0.68	0.60	0.73	0.96	0.94
Welfare	1.26	1.37	1.55	1.67	2.80	2.84
Final emissions per firm	0.43	0.44	0.35	0.43	0.11	0.29
Emissions abated per firm	0.05	0.24	0.25	0.30	0.85	0.65

Table 3.1: SPE outcome for $b = c = s = 1$, $\gamma = 1.5$ and different values for a.

of other variables. Table 3.1 (fifth row) shows that this is not the case when the levels of emissions are determined endogenously as to maximize social welfare. De facto, the socially optimal levels of emissions under both policy instruments are different from each other. This key implication of endogeneity in emission levels is shown to apply also in the next section, where social welfare is maximized assuming a constant value of parameter a.

3.2.3.2 Fixing parameter $a = 2.5$

As in previous section, Proposition 3.4 illustrates the relationship among equilibrium levels of R&D, individual output and social welfare. This time, however, these values are computed for a fixed a.

Proposition 3.4. *Let $b = c = s = 1$ and $a = 2.5$. Then* *a) for* $1.11 < \gamma < 1.18$, $x^e > x^p$ *and* $q^e > q^p$;

b) for $1.18 < \gamma < 1.67$, $x^e > x^p$ *and* $q^p > q^e$;

c) for $1.67 < \gamma$, $x^p > x^e$ *and* $q^p > q^e$;

d) $W^p > W^e$ *for all* $1.11 < \gamma$;

e) e^* *and* h^* *are increasing in* γ;

f) x^e, x^p, q^e, q^p, W^p *and* W^e *are decreasing in* γ *for all* $1.11 < \gamma$[4].

Table 3.2 illustrates the results of the comparison between emission and performance standards for different values of γ. As emphasized previously, endogenizing the level of emissions leads to the differences in the amount of emissions produced under each policy instruments (fifth row of Table 3.2). In essence, however, the welfare result remains unchanged - irrespective of the relationship between the R&D investment and the level of output (part (d) of Proposition 3.4) the society is still wealthier under performance standard. For instance, for $\gamma = 1.80$, performance standard induces a higher R&D ($0.21 > 0.19$), and a higher output ($0.73 > 0.56$). A drop in the value of γ to 1.15 reverses this relationship: a considerably higher level of R&D ($0.81 > 0.43$) and a higher level of output ($0.77 > 0.74$) is attained under emission standard. Yet, the dominance of the welfare level under performance standard is preserved.

[4]The condition $\gamma > 1.11$ is required by (A5), in particular, for $e^* > 0$.

Variable	e.s.	p.s.	e.s.	p.s.	e.s.	p.s.
	($\gamma = 1.15$)		($\gamma = 1.20$)		($\gamma = 1.80$)	
Standard	0.09	0.50	0.17	0.52	0.40	0.62
R&D per firm	0.81	0.43	0.65	0.39	0.19	0.21
Individual output	0.77	0.74	0.72	0.74	0.56	0.73
Welfare	1.64	1.70	1.61	1.69	1.53	1.66
Final emissions per firm	0.09	0.37	0.17	0.38	0.40	0.45
Emissions abated per firm	0.68	0.37	0.55	0.36	0.16	0.28

Table 3.2: SPE outcome for $b = c = s = 1$, $a = 2.5$ and different values for γ.

3.2.3.3 Making the standards e and h exogenous for $a = 2.5$

To better compare the R&D incentives under (endogenous) emission and performance standards with the incentives obtained in the exogenous setting of Montero's (2002) paper, we fix the levels of emission and performance standards to be e and h respectively. Specifically, if q^h is the equilibrium output under the exogenous performance standard h, we assume that $e = q^h h$, so that in equilibrium, the emissions generated by the production process are the same regardless of the instrument.

In this section, we fix h such that $h = h^*$, i.e., such that it corresponds to the endogenous level of the performance standard obtained in Section 3.2.3.2. This gives us $e = q^p h^*$, which implies that the level of emissions will be that given by the endogenous performance standard. Notice that we could fix e and h, so that $e = q^h h$, for any arbitrary value of h, not necessarily the welfare maximizing h^*. The reason we chose the particular value of h^* is to maintain the relevance of endogeneity in our model. In that way we are able to capture the effects that the alteration from endogenous to exogenous e has on the R&D incentives. Hence, we take the outcome of this model and contrast it with the equilibrium values of e and h derived from the maximization of social welfare. We also discuss the relevant implications.

Hence, suppose that the regulator chooses a level of emissions allowed that would optimally result from the three-stage-game under performance standard (all the values in the fifth row of Table 3.2 denoted *p.s.*). The corresponding values of R&D and output are shown in Table 3.3, where the assumptions regarding the values of parameters b, c, s, and a are maintained.

Comparing Tables 3.3 and 3.2 yields interesting insights into the impact of endogeneity in the level of emissions on the equilibrium values of output and R&D. Observe that assuming the level of emissions equal to the one under (endogenous) performance standard necessarily leads to the second, fourth and sixth columns of Table 3.2 and Table 3.3 being identical. The same assumption implies that the first, third and fifth columns of Tables 3.2 and 3.3 do not coincide. In fact, it is easy to see, for instance, that the level of social welfare is (weakly) smaller in Table 3.3. The reason is that $e = q^p h^*$ does not constitute the socially optimal level of emission standard, hence, unlike the level of performance standard, it does not maximize the welfare. In both cases, however, the level of output and the level of R&D investment decreases in γ. The rising cost of R&D discourages the firms from investing into R&D,

Variable	e.s.	p.s.	e.s.	p.s.	e.s.	p.s.
	($\gamma = 1.15$)		($\gamma = 1.20$)		($\gamma = 1.80$)	
Standard (exogenous)	0.37	0.50	0.38	0.52	0.45	0.62
R&D per firm	0.42	0.43	0.37	0.39	0.16	0.21
Individual output	0.64	0.74	0.62	0.74	0.55	0.73
Welfare	1.59	1.70	1.58	1.69	1.53	1.66
Final emissions per firm	0.37	0.37	0.38	0.38	0.45	0.45
Emissions abated per firm	0.27	0.37	0.24	0.36	0.10	0.28

Table 3.3: Outcome for $b = c = s = 1$, $a = 2.5$, different values for γ and exogenous e and $h = h^*$ such that $e = q^p h^*$.

which negatively influences the production of output. This effect combined with the increase in the level of emissions ultimately leads to a lower amount of abatement.

The examination of the levels of R&D investment under both policy instruments yields another interesting result. In particular, for $\gamma = 1.15$ and $\gamma = 1.2$ these levels are no longer higher under emission standard. In fact, Table 3.3 demonstrates that performance standard encourages larger investment in R&D for the entire range of values between 1.15 and 1.80 associated with the cost of R&D, γ. As a result, the amount of emissions abated under performance standard is larger than that under emission standard.

In addition to alterations of the R&D level, the change in the level of emissions from the socially optimal one to $e = q^p h^*$ also affects the level of output. When the emission levels are endogenous (Table 3.2), a larger output is produced under emission standard assuming that $\gamma = 1.15$. However, for $\gamma > 1.15$ it is the performance standard that induces more output. This reversal does not take place under the assumption of exogenous e (Table 3.3). In fact the level of output under performance standard hardly varies with γ and remains larger than the output level produced under emission standard for all γ. Consequently, one can derive the conclusion that fixing the level of emission standard to be $e = q^p h^*$ (the socially optimal level of emissions under performance standard) is the reason why the induced level of output is lower under emission standard [5].

Next, we contrast these results with those provided by Montero (2002).

3.2.3.4 Discussion

In his paper, Montero (2002) compares the incentives to invest in the environmental R&D under emission and performance standards by assessing the induced direct and strategic effects under each policy instrument. On one hand, he establishes that the direct effect is larger under emission standard. This is because the investment in the environmental R&D causes the level of output and emissions to increase less under emission standard than under performance standard. Consequently, it is possible to abate a larger amount with emission constraint that is fixed, than with a performance standard that varies with the level of output. This is the direct or cost-minimizing

[5] In order to induce the level of output that is higher under emission standard than under performance standard, the corresponding level of emissions per firm must be sufficiently low.

effect. On the other hand, the same increase in output is responsible for the dominance of the strategic effect under performance standard. In particular, the flexibility associated with the output variations under performance standard enables the level of output to grow faster, which, subsequently, leads to the advantage in the value of profit earned. Therefore, taking the direct and strategic effects together gives no clear indication regarding which of the two instruments provides more incentives to invest in the environmental R&D.

This paper extends Montero's (2002) comparison of the R&D incentives under emission and performance standards by accounting for welfare effects. As shown in Tables 3.1 and 3.2, the maximization of social welfare entails endogenizing the levels of emissions under each policy instrument. This endogeneity in the level of emissions creates some similarities and differences with the model of Montero. We discuss the similarities first.

The increase in the value of parameter γ (see Table 3.2) implies that the marginal cost of R&D investment increases as well. The latter one, in turn, negatively affects the level of production, which, subsequently, drops. This effect coupled with simultaneous rise in the level of emissions allowed per firm leads to an overall fall in the amount of the emissions abated. Since the level of emissions that is allowed is considerably higher under performance standard, the number of emissions that require abatement is much lower. The reason for this is that, unlike emission standard, performance standard enables a better adjustment of the emission to output level leading to a lower emission abatement required under the latter policy instrument.

This implies, however, that the compliance cost under emission standard is significantly higher. Consequently, one has more incentive to invest in R&D and to reduce the abatement cost under emission standard. Thus, this effect, known as the direct effect in Montero's framework, leads to the higher R&D incentives under emission standard.

At first sight, it might seem that our analysis of R&D incentives supports the findings of Montero (2002). However, no such claim can be made as long as the underlying values of emissions are different in both models. Hence, we fix the level of emission standard e (see Table 3.3) to match that of the socially optimal performance standard, $q^p h^*$, where q^p is the equilibrium output under the endogenous performance standard h^*. In that way, we are able to compare the R&D incentives under both policy instruments with those in Montero's model.

For $e = q^p h^*$ performance standard provides a larger R&D investment. This is different to the case where e is endogenous, in which more R&D is invested under emission standard for $\gamma = 1.15$ and $\gamma = 1.2$. The fact that more R&D investment is undertaken under performance standard raise a doubt about Montero's (2002) conclusion regarding the comparison of direct effects under both policy instruments. Montero emphasizes that for positive values of R&D investment the direct effect is larger under emission standard because the corresponding abatement level is greater. It is clear from Table 3.3 that the contrasting result is obtained: for an equal level of emissions in both regimes, the level of output is lower under emission standard leading to a lower level of abatement. As a result, the R&D incentives under emission standard are lower. Thus, we establish that Montero's direct effect has opposing implications in our framework, where $e = q^p h^*$. In particular, the positive R&D investment is neither sufficient to imply a higher level of abatement nor to imply more R&D incentives.

Another important insight into the comparison of R&D incentives under both policy instruments concerns the level of output. In Section 3.2.3.3 we have shown that fixing e changes the qualitative implications regarding the level of output under both standards. In particular, the level of output under performance standard becomes consistently higher and hardly varies with parameter γ (Table 3.3, third row). This observation contradicts earlier findings of Montero (2002), who recognizes that a rise in R&D investment causes the output level to increase more rapidly under performance standard. According to this description of the strategic effect, a decreasing R&D investment should imply a more rapid decrease in the output level under performance standard. This is neither the case when e is endogenous (Table 3.2) nor when it is fixed (Table 3.3). Hence, the insensitivity of the output level under performance standard to the alterations in the level of R&D indicate that the level of rival's output remains unaffected. Consequently, the strategic effect as defined by Montero (2002) does not take place in the current analysis, leaving other effects to influence the R&D incentives.

In summary, we provide the comparative analysis of R&D incentives under emission and performance standards. Such analysis has been previously undertaken by Montero (2002), who, however, did not account for the maximization of social welfare. As a result, his conclusions are derived for fixed values of emissions under both policy instruments. We revisit the debate on R&D incentives in this model by looking at the socially optimal levels of emissions under both policy instruments. Unlike Montero, our method of comparison of R&D incentives is not based on the assessment of the direct and strategic effects. In fact, we find little evidence for the presence of these effects in our framework. For instance, we establish that performance standard provides larger R&D incentives than emission standard (Table 3.3). This contradiction to the results that Montero (2002) can be explained by noting that in our model the level of output under performance standard is larger, implying a higher level of emission abatement. As a result, in order to reduce the abatement cost a firm has more incentive to invest in R&D under performance standard.

Our model provides even less evidence of the strategic effect. Marginal changes in the quantity of output being produced under performance standard are insufficient to justify the presence of strategic effect as defined by Montero (2002). In particular, the change in output induced by the alteration in the cost of R&D is so small that it affects the output of the rival to a minimal extent. Hence, the strategic component of R&D in our model is absent.

By evaluating the results obtained in Tables 3.1, 3.2 and 3.3, only a single unambiguous result can be identified - the level of social welfare under performance standard is higher. This result holds irrespective of which instrument induces a higher level of output, a higher level of emissions or a higher level of abatement. In fact, this is the only result that does not depend on the investment in R&D. Nevertheless, the independence of the R&D investment, makes it no longer justifiable to claim that the instrument which implies a higher level of social welfare is also the one that provides larger R&D incentives. In particular, the comparison of R&D incentives is driven by two contradicting effects. One is the effect of endogeneity of emission levels under which emission standard dominates in the provision of R&D incentives (Table 3.2). The other effect concerns the comparison of R&D incentives for the same level of emissions (Table 3.3). With regards to the latter one, performance standard is superior. Hence, the ultimate result of the comparison between R&D incentives induced under emission and performance standards is the combination of the two effects, whose closer form cannot be established.

3.3 Conclusion

This paper extends the analysis of R&D incentives under emission and performance standards of Montero (2002) to account for welfare effects. In particular, it examines which of the two policy instruments provides larger incentives to invest in R&D in the model, in which social welfare is maximized. While larger welfare is obtained under performance standard, the level of R&D incentives is driven by two opposing effects. On one hand, emission standard is superior when the levels of emissions under both policy instruments are determined endogenously. On the other hand, performance standard dominates when the emissions levels are equal for both standards. Consequently, determining which instrument yields larger R&D incentives depends not only on the primitives on the model, but also on the nature of the emissions levels under emission and performance standards. Finding a way of comparing these contradictory effects in a general setting would be an interesting avenue for future research.

3.4 Proofs

Proof of Proposition 3.1. In the last stage, firm i solves the optimization problem (3.2), which leads to the following first-order condition (FOC)

$$-2bq_i + a - bq_j - c + x_i = 0, \tag{3.8}$$

thus, the reaction function of firm i is

$$q_i(q_j, x_i) = \begin{cases} \frac{a-c+x_i}{2b} - \frac{q_j}{2}, & \text{if } \frac{a-c+x_i}{b} \geq q_j \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, the objective function is strictly concave, thus, we have a maximum (the second-order condition (SOC) is $-2b < 0$). Solving simultaneously the two non-zero reaction functions of the firms we get that in equilibrium

$$q_i(x_i, x_j) = \frac{a - c + 2x_i - x_j}{3b}, \tag{3.9}$$

for $i \neq j$.

In the second stage, firm i chooses its level of R&D, x_i, given x_j, e and $q_i(x_i, x_j)$ in equation (3.9).

The FOC of (3.3) is then given by

$$\frac{2}{3b}[-2bq_i(x_i, x_j) + a - bq_j(x_i, x_j) - c + x_i] + \frac{4}{3}q_i(x_i, x_j) - e - \gamma x_i = 0. \tag{3.10}$$

Notice that the first term in the previous equation vanishes by FOC (3.8). By (A5), $9b\gamma > 8$, the SOC holds. Plugging equation (3.9) into equation (3.10) and using the fact that the firms are symmetric and subject to the same standard e, we get that every firm invests

$$x(e) = \frac{4(a-c) - 9be}{9b\gamma - 4} \tag{3.11}$$

in R&D, with production

$$q(e) = \frac{3\gamma(a-c) - 3e}{9b\gamma - 4}. \tag{3.12}$$

Finally, we solve the problem of the regulator, i.e., problem (3.4). Here, the FOC is given by the following equation.

$$\frac{dq(e)}{de}[a - 2bq(e) - c + x(e)] + \frac{dx(e)}{de}[q(e) - e - \gamma x(e)] + c - x(e) - 2se = 0. \tag{3.13}$$

The SOC of this problem is $\frac{2(9b+8s-18b\gamma s)}{9b\gamma-4}$, which is negative by (A5): $9b\gamma > 8$ and $9b + 8s < 18b\gamma s$. Using the FOC's (3.8) and (3.10), equation (3.13) becomes

$$bq(e)\frac{dq(e)}{de} - \frac{1}{3}q(e)\frac{dx(e)}{de} + c - x(e) - 2se = 0. \tag{3.14}$$

Equations (3.11), (3.12) and (3.14) together, lead to the results in the proposition. (A5) guarantees that in equilibrium, $0 < x^* < c$ and $0 < e^* < q^*$. □

Proof of Proposition 3.2. The FOC of problem (3.5) is given by

$$a - 2bq_i - bq_j - (c - x_i)(1 - h) = 0, \tag{3.15}$$

with SOC $-2b < 0$.

Solving simultaneously for the positive parts of the reaction curves of the firms, leads to

$$q_i(x_i, x_j, h) = \frac{(2x_i - x_j - c)(1 - h) + a}{3b}. \tag{3.16}$$

Using equation (3.16) in problem (3.6) drives the following FOC.

$$\frac{2}{3b}(1-h)[a-2bq_i(x_i,x_j,h)-bq_j(x_i,x_j,h)-(c-x_i)(1-h)]+\frac{4}{3}q_i(x_i,x_j,h)(1-h)-\gamma x_i=0, \tag{3.17}$$

with SOC $8(1-h)^2/(9b)-\gamma<0$, which holds by (A6).

The first term in the previous equation vanishes by FOC (3.15). Equations (3.16) and (3.17), plus symmetry of the model imply the result. (A6) guarantees that $0<x(h)<c$ and $q(h)>0$. □

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